THE COST OF CAPITAL TO ELECTRIC UTILITIES: A DIRECT ESTIMATION

BY

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To my wife, Kathy,
and our sons,
John and Ben

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The focus of the dissertation is upon a model of firm total value which allows for the direct calculation of the average cost of capital to the firm. Particular emphasis is placed upon empirical vertification of the model by a nonlinear regression technique. An investigation into the equivalent risk class assumption is made which rejects the notion that industrial classification assures the selection of firms homogeneous in business risk. An ad hoc procedure to select firms of equal business risk is undertaken and a casual test reveals that financial leverage has no significant impact on firm total value.

Chapter II examines the partial equilibrium theory of the cost of capital. Chapter III reviews several prominent empirical cost of capital studies. Chapter IV develops the direct cost of

capital model and is followed by empirical verification in Chapter V. The model provides reasonable estimates of the average cost of capital.

CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

Throughout their major empirical study of the cost of capital to the electric utility industry Merton Miller and Franco Modigliani [39] defined the term cost of capital as "that minimum prospective rate of yield that a proposed investment in real assets must offer to be just worthwhile undertaking from the standpoint of the current owner of the firm."[1] Such a definition for cost of capital is largely unopposed. It is equivalent to "the rate of yield that a proposed investment in real assets must offer to be just worthwhile undertaking to leave the total market value of the firm unchanged."[2] Because the rate of yield noted in these definitions is impossible to observe, it follows that the average cost of capital is also unobservable. Financial economists, realizing this limitation, can only attempt to make inferences about the cost of capital. They base their attempts on what they can observe: (1) the current market value of the firm, and (2) their expectations about the future stream of benefits that existing security holders expect to receive.

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[1] Miller and Modigliani [39, p. 335].

[2] Haley and Schall [27, p. 218].

Disagreement tends to develop at this point. That is, how should the benefits relative to market values for a variety of securities be combined to form an overall cost of capital for the firm? The purpose of this study is to propose and empirically test an effective method for estimating directly the overall cost of capital. The term directly is emphasized because the calculating equation, unlike models of the past, allows one to by-pass two thorny issues: (1) the assignment of weights to the several component costs, and (2) the correct specification of the after-tax component cost of debt. In the direct model (proposed in Chapter IV), the cost and relative importance of each component of capital make their contribution to and are reflected in the total market value of the firm.

Previous cost of capital studies are plentiful; many are referenced in subsequent chapters. However, they are principally concerned with the cost of equity capital and the impact of financial leverage on firm value. The average cost of capital is calculated only incidentally, if at all. Miller and Modigliani [39] do make use of the total value approach, but a specification error concerning the tax subsidy effect (discussed in Chapter II) tends to raise questions about their results. So far as this writer is aware no one has estimated the average cost of capital directly, i.e., without first calculating the several component costs.

The Sample

To empirically verify the direct cost of capital model with respect to financial leverage, a sample of firms homogeneous in their degree of business risk is required. As will become apparent, risk-equivalent firms in the strictest sense of the definition are rare. In most of the studies to date (cf. [26], [32], [36], [39], [41], and [54]) researchers testing cost of capital propositions have been content to select firms from the same industry. Often, electric utilities have been selected as a sample based on the belief that electric utilities are probably less heterogeneous than other industries. That industrial classification in general fails to provide firms of equal risk has been shown by both Wippern [56] and Gonedes [23], and is convincingly demonstrated in Appendix B of this study. The firms selected to verify the proposed valuation model are electric utilities. Rather than relying strictly on industrial classification, an attempt to refine the sample to more precisely satisfy the theoretical definitions of equivalent business risk is undertaken. It is only partially successful.

There are other reasons for selecting electric utilities as the sample, not the least of which was a grant to do so. The importance for regulated industries to have a good estimate of their cost of capital has taken on increased significance in regulatory hearings. Because electric utilities are regulated by their rate of return, they must periodically present evidence demonstrating what level of return they should be permitted to earn on their base of income generating assets. The average cost of capital is increasingly being used as an appropriate benchmark on which to base a utility's allowed rate of return.

The Regulatory Setting

There is no argument, at least at the constitutional level, about what rate of return a public utility should earn on the capital provided by its common stockholders. Since 1944 the legal standard has been unaltered.

> The return to the equity owner should be commensurate with returns on investments in other enterprises having corresponding risks. That return, moreover, should be sufficient to assure confidence in the financial integrity of the enterprise, so as to maintain its credit and to attract capital.[3]

This important statement by the Supreme Court, while evidently quite satisfactory in a judicial sense, has yet to receive a unanimous interpretation among participants of the regulatory process.

In general, the objective of a regulatory agency is to determine a price for the utility's service, sufficiently above the operating cost of producing that service, so that the utility earns the lowest rate of return on its capital consistent with investment in new facilities required by the public interest. Such an objective has something in common with the cost of capital. Indeed many expert witnesses who participate in regulatory proceedings have advocated allowing a utility to just earn its cost of capital. The estimation process for arriving at a cost of capital number may vary slightly with different witnesses and different states, but the methods are for the most part consistent. First, embedded (historical) costs for existing debt and preferred stock are calculated, followed by an estimation of the required rate of return on equity by the Gordon constant growth model, the Sharpe-Lintner capital asset pricing model, or a comparison with current equity yields for "similar types" of firms. Then these component costs are combined into a weighted average and labeled the cost of capital to the utility. At the outset of this study it must be emphasized that this average number, regardless of its label, is not a firm's cost of capital. A simplified equation for the calculation (referred to hereafter as allowed rate of return) is

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[3] FPC v. Hope Natural Gas Company, 320 U.S. 591, 603 (1944).

$$Z_a = k_a \frac{E}{RB} + (1 - \tau)e \frac{B}{RB} + p \frac{PF}{RB} \tag{1.1}$$

where

Z_a = the net of tax rate of return the utility is permitted to earn on its rate base

k_a = the after-tax allowed rate of return on the utility's common equity

e = the average embedded pre-tax cost of the utility's bonds

p = the average embedded after-tax cost of the utility's preferred stock

RB = the firm's rate base assumed to equal E + B + PF

E = the total book (recorded) value of the utility's common equity

B = the total book value of the utility's debt

PF = the total book value of the utility's preferred stock

τ = the firm's tax rate

The structure of equation (1.1) is quite similar to the weighted average cost of capital found in leading finance textbooks (cf. [55] and [53]). However, it is not the same. The values of k_a, e, p, E, B, and PF are established by a quasi-judicial authority. The average cost of capital, on the other hand, is determined by competitive forces in a relatively perfect capital market. Equality between equation (1.1) and the firm's cost of capital is possible; perhaps it is desirable. That the cost of capital to a utility should influence the utility's allowed rate of return is suggested.

An example of an actual company's allowed rate of return calculation may be helpful in understanding the regulatory setting. In late 1972 Florida Power Company petitioned the Florida Public Service Commission for authority to increase the rates it charged to its customers. A portion of that hearing centered around the issue of allowable rate of return. After determining the allowed capital structure for Florida Power, attention was turned to the costs of each component included in the capital structure. For both debt and preferred stock, average embedded costs of 5.80 percent and 6.11 percent respectively were allowed. The cost of common equity was more complicated. The company through their expert witness requested 15.0 to 16.0 percent. The Atomic Energy Commission through its own witness suggested 11.8 percent. The Commission decided that a fair and reasonable return on equity should be within a range of 13.50 and 14.25 percent. With customer deposits and tax credits included in the capital structure the weighted allowed rate of return was calculated as shown in Table 1.1. The Commission set 8.22 percent as the allowed rate of return to be applied to the rate base.

Just as important as the allowed rate of return in determining the total amount of allowable revenues a utility can receive is the rate base. Different states value the rate base differently, the possibilities varying from 100 percent historical (original) cost to 100 percent replacement cost. Texas recently ruled that the rate base for Southwestern Bell would be valued at 40 percent historical cost

Table 1.1

Cost of Capital Calculation for Florida Power Corporation

Class of Capital	Percent	Cost (%)	Weighted Component (%)
Long-term Debt	50.0	5.80	2.90
Preferred Stock	7.1	6.11	.43
Common Equity	35.1	13.5-14.25	4.75-5.00
Investment Tax Credit (3%) and Tax Deferrals	6.8		
Customer Deposits	1.0	6.00	.06
Total	100.0		8.13-8.39

Source: Docket No. 71370-EU, Order No. 5619 December 29, 1972; 98 *Public Utility Report* 3rd volume.

and 60 percent replacement cost. No reason was provided. Most states, however, rely heavily on historical cost.

Excess Return and Economic Efficiency

It is not the purpose of this study to provide support to existing proposals that a utility be allowed to earn its cost of capital and no more. However, a brief summary of the arguments for and against returns in excess of the cost of capital may be of some worth to readers. First, the arguments for allowing excess return generally follow the notion that without the possibility of excess returns there is little if any incentive for the firm to be efficient in its choice of technology, capacity, price and output. Also, if by law the utility is not able to earn more than its cost of capital, then a rate of return petition will occur whenever there is a change in the cost of a capital component. The end result: consumers, not investors, bear greater risk. Thus, it would benefit consumers through greater efficiency and reduced regulatory costs to allow a return somewhat in excess of the cost of capital.

On the other hand, Averch and Johnson [9] have shown that whenever a utility can expect to earn a return in excess of the cost of capital, an incentive exists for such a firm to use more than an efficient amount of capital relative to other factors of production. Furthermore, Baumol and Klevorick [11] have shown that as the difference between the cost of capital and expected allowed return decreases, the inefficiency in factor proportions actually increases.

A compromise between the two extreme positions is to consciously employ a lag between regulatory hearings. During the lag the utility can enjoy high, or suffer low, profits for short periods of time (Myers [45, p. 82]). But regardless of whether or not a regulatory agency allows a utility to earn a return equal to or greater than the cost of capital, and whether or not regulation is continuous or administered with a conscious lag, it seems logical that rate of return regulation have as a beginning point the overall cost of capital.

Hypotheses to be Tested

The major hypothesis to be tested is that the cost of capital to a firm can be estimated by applying the constant growth valuation model to the firm as an entity rather than the traditional approach of using the growth model only to estimate the cost of the firm's equity, then combining the estimated cost of equity with other component costs by some form of a weighted average. Because the growth component of the model is nonlinear, an iterative least squares procedure is used to empirically test the model. In this way the nonlinearity in the model's growth component is explicitly recognized. To provide some evidence that the valuation model is correctly specified, a casual test for the impact of financial leverage on firm value is undertaken. The minor hypothesis tested is that financial leverage has no impact on the total firm value of electric utilities. To test for this impact however, there must be some assurance that the test sample is homogeneous in the level of nonfinancial risk exhibited by the firm. That electric utilities in general form such a risk class is the second minor hypothesis that is tested.

Organization of the Study

This chapter has presented an introduction to the development of a direct estimating procedure for the average cost of capital, a brief review of the regulatory environment, and a specific statement of the hypotheses to be tested. Chapter II reviews theoretical cost of capital specifications, the robustness of those specifications and the misspecification of the Miller and Modigliani growth model. Chapter III reviews several empirical investigations of firm valuation and cost of capital. In Chapter IV the valuation model is developed. Properties of the model are examined and illustrated graphically. Also a procedure for adjusting the allowed rate of return is proposed. Chapter V presents empirical results of cross-section regressions for the major and minor hypotheses. A summary of findings and areas for future research are presented in Chapter VI.

CHAPTER II

REVIEW OF CURRENT PARTIAL EQUILIBRIUM COST OF CAPITAL MODELS

In Chapter I the average cost of capital to a firm was defined as the discount rate that equates the capitalized value of a firm's expected cash flows to its current market value. In spite of this seemingly straightforward definition dissimilar cost of capital specifications exist. The two most common specifications are the traditional model and the Modigliani and Miller (hereafter M & M) model. Underlying assumptions about firm and market characteristics have been used to explain differences between the two models (Myers [46]). But even within the assumptions of the traditional model, different cost of capital specifications obtain depending upon the definition of cash flow (cf. Arditti [4], Beranek [12], Nantell and Carlson [47], and Arditti and Levy [7]).

The purpose of Chapter II is to: (1) set forth the several specifications of average and marginal costs of capital; (2) examine the robustness of the M & M and "traditional" marginal cost of capital specifications; (3) correct the misspecified M & M valuation equation for the levered firm with growth opportunities; and (4) set forth acceptable conditions for firms to be considered equal in business risk.

Assumptions

A consistent analysis will be aided by the following assumptions. In a later section of the chapter these assumptions will be examined for sufficiency in deriving the M & M and traditional models.

1. Dividend policy is irrelevant.
2. Investment projects are perpetuities.
3. Investment projects make a permanent contribution to debt capacity.
4. Acceptance of an investment project does not shift a firm's target debt ratio.
5. Investment projects do not change a firm's business risk class.
6. Financial leverage is irrelevant except for corporate income taxes.
7. The firm's assets are expected to generate a constant and perpetual earnings stream.
8. The firm is already at its target debt ratio.

Defining Cash Flow

Let $\overline{CR}(t)$ be defined as the expected cash receipts to the firm and $\overline{CD}(t)$ as the expected cash disbursements except for interest payments in period t. The difference between $\overline{CR}$ and $\overline{CD}$ is $\overline{CF^{\tau}}$, the net cash flow to creditors and owners of the firm. The subscript t has been omitted to indicate a constant expected net cash flow over the economic life of the asset. The superscript τ is hereafter used to designate a flow variable net of taxes. Net cash flow available to investors can be described in either of two ways: (1) as an infinite stream equal to the sum of earnings available to stockholders plus interest payments, or (2) as a cash flow, the duration of which is equal to the economic life of the firm's assets. The latter case is expressed as:[1]

$$\overline{CF}^{\tau} = (\overline{X} - iD)(1 - \tau) + iD + Dp \qquad (2.1)$$

where

$\overline{X}$ = expected net operating income or earnings before interest and taxes but after deduction of economic depreciation

i = yield to maturity on the debt of the firm

D = market value of the firm's total debt

τ = the corporate tax rate

Dp = economic depreciation

The total money expected to be received by common stockholders is $(\overline{X} - iD)(1 - \tau)$. The total dollar interest payment to be received by holders of the firm's debt is iD. The two payments can be conveniently combined as the expected ultimate income stream to investors

$$\overline{X}^{\tau} = (\overline{X} - iD)(1 - \tau) + iD \qquad (2.2)$$

or

$$\overline{X}^{\tau} = \overline{X}(1 - \tau) + \tau iD, \qquad (2.2a)$$

giving an expected ultimate net cash flow of

$$\overline{CF}^{\tau}_{uf} = \overline{X}^{\tau} + Dp \qquad (2.3)$$

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[1] Throughout this chapter only the debt and equity components of capital will be considered. Inclusion of preferred stock in the analysis enlarges the equations without contributing to the analysis.

Capitalizing Net Cash Flow

Plant and equipment has a finite life. Therefore, it seems intuitively obvious that the cash flow described by (2.3) can last no longer than this economic life, and to provide an income stream for more years than the economic life of existing assets requires additional investment in the firm. This additional investment is economic depreciation. It is defined as the amount of dollars Dp, which invested for N years at the cost of capital Z, will assure the continuation of the existing level of cash flow $\overline{X}^{\tau}$ + Dp. That is,

$$Dp(1 + Z)^N = \overline{X}^{\tau} + Dp \tag{2.4}$$

It makes absolutely no difference whether investors capitalize net cash flow over N periods, the economic life of the firm's existing assets, or if the ultimate payments to investors--when those payments are assumed to last forever--are discounted over an infinite life. Capitalizing $\overline{CF}^{\tau}_{uf}$ over N periods gives

$$V_{uf} = \sum_{t=1}^{N} \frac{\overline{CF}^{\tau}_{uf}}{(1 + Z)^t} = \frac{\overline{CF}^{\tau}_{uf}}{Z} - \frac{\overline{CF}^{\tau}_{uf}}{Z(1 + Z)^N} \tag{2.5}$$

where V_{uf} is total firm value for a firm with cash flow defined by (2.3).

Substituting $\overline{X}^{\tau}$ + Dp for $\overline{CF}^{\tau}_{uf}$ in equation (2.5) and $(\overline{X}^{\tau} + Dp)/(1 + Z)^N$ for Dp results in

$$V_{uf} = \frac{\overline{X}^{\tau}}{Z} \tag{2.6}$$

But (2.6) is equal precisely to

$$V_{uf} = \sum_{t=1}^{\infty} \frac{\overline{X}^{\tau}}{(1 + Z)^{t}} = \frac{\overline{X}^{\tau}}{Z} \tag{2.7}$$

An Alternative Cash Flow Concept

Equation (2.2), the expression describing the ultimate flow of cash to investors, represents dollars that the suppliers of the firm's capital expect to receive as periodic payment for providing funds to the company. Another way to view cash flow is to begin with operating income--earnings before interest and taxes--and note that the unlevered firm can be expected to pay a periodic dividend of $\overline{X}(1 - \tau)$. If the firm finances a portion of its assets with debt, the expected payment to equity holders is reduced by $(1 - \tau)iD$. However, the effective after-tax interest payment to bondholders is also $(1 - \tau)iD$, exactly offsetting the reduced expected income to equity holders. Thus, the expected net cash flow can be written as

$$\overline{CF^{\tau}_{oi}} = (\overline{X} - iD)(1 - \tau) + (1 - \tau)iD + Dp \tag{2.8}$$

$$\overline{CF^{\tau}_{oi}} = \overline{X}(1 - \tau) + Dp \tag{2.8a}$$

As in the previous analysis (see equations (2.4) through (2.7)) identical numbers obtain whether $\overline{CF^{\tau}_{oi}}$ is discounted over the life of existing assets, or if $[(\overline{X} - iD)(1 - \tau) + (1 - \tau)iD]$ is capitalized over an infinite number of periods. In either case the result of capitalizing net operating income is

$$V_{oi} = \frac{\overline{X}(1 - \tau)}{Z} \tag{2.9}$$

where

V_{oi} = the derived value of the firms.

It is not to be implied that identical numbers would obtain for V if $\overline{CF}^{\tau}_{uf}$ and $\overline{CF}^{\tau}_{oi}$ were each discounted at the same rate Z for N periods. They would not. $\overline{CF}^{\tau}_{uf}$ would always provide a larger value for V since $\overline{CF}^{\tau}_{uf} - \overline{CF}^{\tau}_{oi} = \tau iD$.

The Average Cost of Capital

In this section, three average cost of capital specifications are reviewed.[2] The first one is the traditional model, often referred to as the textbook formula. Although all three specifications can correctly be described as weighted averages, the traditional model is the one that usually comes to mind when reference is made to the weighted average cost of capital. The validity of the traditional model rests upon the correctness of net operating income (see equation (2.8)) as the cash flow to be capitalized. The second and third models reviewed rely upon the ultimate flow to investors' definition of cash flow (see equation (2.1)). Arditti and Levy [7] have argued convincingly that the latter leads to the correct specification of the cost of capital.

Traditional Specification

Assuming cash flow perpetuities, the average cost of capital for the unlevered firm is

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[2] Omitted from this review are cost of capital specifications for a tax-free world. Two extremes to valuation in a tax-free world are contained in Durrand [19].

$$Z_T = \frac{\overline{X}(1 - \tau)}{V} \tag{2.10}$$

where

V = total market value of the firm

Z_T = the discount rate that equates V and $\overline{X}(1 - \tau)$, i.e., the cost of capital

For the firm that finances its assets in part by debt,

$$Z_T = \frac{(\overline{X} - iD)(1 - \tau)}{V} + \frac{i(1 - \tau)D}{V} \tag{2.11}$$

Multiplying the first term on the right-hand side of equation (2.11) by S/S where S is the market value of the firm's common stock, and defining

$$k = \frac{(\overline{X} - iD)(1 - \tau)}{S} \tag{2.12}$$

as the required after-tax rate of return on equity gives

$$Z_T = k\frac{S}{V} + i(1 - \tau)\frac{D}{V} \tag{2.13}$$

Equation (2.13) expresses the average cost of capital as the sum of the required after-tax yields on equity and debt, weighted by their respective market determined proportions of total value.

Ultimate Flow Specification

Defining cash flow by equation (2.3) provides an average cost of capital of

$$Z_{UF} = \frac{(\overline{X} - iD)(1 - \tau) + iD}{V} \tag{2.14}$$

Once again multiplying $(\overline{X} - iD)(1 - \tau)$ by S/S and recognizing that the required rate of return on equity is defined as (2.12), results in average cost of capital specified as

$$Z_{UF} = k\frac{S}{V} + i\frac{D}{V} \qquad (2.15)$$

Comparison of Z_T and Z_{UF}. Comparing equations (2.15) and (2.13) reveals that

$$Z_{UF} = Z_T + i\tau\frac{D}{V} \qquad (2.16)$$

But it should be equally clear that capitalizing the perpetual income stream $(\overline{X} - iD)(1 - \tau) + iD(1 - \tau)$ by rate Z_T will yield a value V exactly equal to the value obtained when perpetuity

$$(\overline{X} - iD)(1 - \tau) + iD$$

is capitalized at the rate $Z_{UF} = Z_T + \tau D/V$.

Modigliani and Miller Specification

Modigliani and Miller [41] and [42] derive propositions for the valuation of the firm. Almost all empirical cost of capital studies refer to these propositions. Indeed, several of the studies reviewed in the next chapter use M & M's Proposition II (reviewed below) as a starting point. M & M initially presented the propositions under the condition of no corporate income taxes. Subsequently, they demonstrated the consequences of tax regulations on the use of debt capital by the firm. It has been argued that because embedded interest costs are tax adjusted in the calculation of a utility's allowed rate of return, the M & M after-tax model overstates the value of electric utilities. Additional reference to this criticism is contained in Chapter III.

In the absence of corporate income taxes M & M set forth the following two propositions:

Proposition I: The average cost of capital to any firm is completely independent of its capital structure and is equal to the capitalization rate for a pure equity stream of the firm's class of business risk. Thus,

$$V \equiv S + D = \frac{\overline{X}}{\rho} \tag{2.17}$$

where

ρ = the market capitalization rate for an unlevered firm of a given class of business risk

Assuming equivalence between corporate and personal financial leverage and the possibility of arbitrage in a perfect capital market, the proof that equation (2.17) represents a state of firm equilibrium in the capital market is straightforward.[3]

Proposition II: The expected yield on the stock of a company belonging to a unique class of business risk is a linear function of financial leverage. Proposition II follows directly from Proposition I. Defining the expected income available to common stockholders in a levered firm as $\overline{\Pi} \equiv \overline{X} - iD$, the expected yield on common stock is

$$\frac{\overline{\Pi}}{S} = \frac{\overline{X} - iD}{S} \tag{2.18}$$

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[3] See Modigliani and Miller [43] or Fama and Miller [22].

Substituting $\rho(S + D) \equiv \rho V$ (see equation (2.17)) for $\overline{X}$ in (2.18) reveals the linear relationship between expected yield on common stock and financial leverage

$$\frac{\overline{\Pi}}{S} = \rho + (\rho - i)\frac{D}{S} \tag{2.19}$$

When taxes are included in the analysis, M & M demonstrate that the presence of debt in the capital structure increases the value of the firm. Total firm value

$$V = \frac{\overline{X}(1 - \tau)}{\rho^{\tau}} + \tau D \tag{2.20}$$

is found by capitalizing each component of the income stream defined in equation (2.2a) by a discount rate appropriate for that component, i.e., $\overline{X}(1 - \tau)$ by ρ^{τ} and τiD by i.[4] Notice that $D = 0$ reduces the valuation equation to the capitalization of the pure equity stream resulting in $V = V_U$, where V_U is the value of an unlevered firm. An alternative expression for equation (2.20) then is

$$V = V_U + \tau D \tag{2.21}$$

After-tax earnings yield. M & M derive an after-tax earnings yield similar in structure to a weighted average cost of capital expression. The earnings yield is defined as the ratio of earnings available to common shareholders plus interest payments to total firm market value, i.e., $\overline{X}^{\tau}/V$. It is derived from equation (2.20) by substituting $\overline{X}^{\tau} - \tau iD$ for $\overline{X}(1 - \tau)$ and solving the resulting equation for

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[4] The proof that in a perfectly competitive market it is possible to discount a risky and riskless component of an income stream by appropriate individual rates can be found in Arditti [5].

$$\frac{\overline{X}^{\tau}}{V} \equiv Z_m = \rho^{\tau} - (\rho^{\tau} - i)\tau\frac{D}{V} \qquad (2.22)$$

Note that D/V = 0 results in $Z_m = \rho^{\tau}$, the after-tax required return for the unlevered firm. As D/V increases, Z_m declines by the constant $(\rho^{\tau} - i)\tau$ over the entire range of D/V, i.e., a corner solution obtains indicating that the value of the firm is maximized when the firm employs all debt as its capital.

After-tax return on common stock. The after-tax required rate of return on common stock is also derived from equation (2.20). First, define

$$\frac{\overline{\Pi}^{\tau}}{S} \equiv \frac{(\overline{X} - iD)(1 - \tau)}{S} \qquad (2.23)$$

as the expected after-tax required return on shares of the firm. Adding and subtracting $iD(1 - \tau)$ to the pure equity term in equation (2.20) and replacing V with S + D gives

$$S + D = \frac{(\overline{X} - iD)(1 - \tau) + iD(1 - \tau)}{\rho^{\tau}} + \tau D \qquad (2.24)$$

Multiplying (2.24) by ρ^{τ}, dividing by S, and combining terms results in the after-tax required rate of return on common stock expressed as a linear function of the debit-to-equity ratio

$$\frac{\overline{\Pi}^{\tau}}{S} = \rho^{\tau} + (1 - \tau)(\rho^{\tau} - i)\frac{D}{S} \qquad (2.25)$$

Summary of Average Cost Formulae

Five general average cost formulae have been reviewed. They are:

Traditional average cost of capital.

$$Z_T = k\frac{S}{V} + i(1 - \tau)\frac{D}{V} \tag{2.13}$$

Ultimate flow average cost of capital.

$$Z_{UF} = k\frac{S}{V} + i\frac{D}{V} \tag{2.15}$$

Modigliani and Miller average cost of capital.

$$Z_m \equiv \frac{\overline{X}^\tau}{V} = \rho^\tau - (\rho^\tau - i)\tau\frac{D}{V} \tag{2.22}$$

Modigliani and Miller required return on common stock (no-tax model).

$$\frac{\overline{\Pi}}{S} = \rho + (\rho - i)\frac{D}{S} \tag{2.19}$$

Modigliani and Miller required return on common stock (tax model).

$$\frac{\overline{\Pi}^\tau}{S} = \rho^\tau + (1 - \tau)(\rho^\tau - i)\frac{D}{S} \tag{2.25}$$

Marginal Cost of Capital

The marginal cost of capital is defined as the rate of return that must be earned on an additional dollar of investment so as to leave the market value of common stock unchanged. If investment in the firm is increased by dI dollars, it follows that the resulting change in firm value is $dV = dS_o + dD_o + dI$, where dS_o and dD_o represent changes in the market values of existing amounts of common stock and debt. Now assuming that acceptance of a particular investment project has no impact on existing debt (see assumptions 4 and 5), then $dD_o = 0$. The definition of marginal cost of capital requires that $dS_o = 0$. Thus, $dV = dI$, or $dV/dI = 1$ is the necessary condition for deriving marginal cost of capital formulae.

Traditional Model

The marginal cost of capital when cash flow is defined as net operating income (see equation (2.8)) is found by differentiating equation (2.9) with respect to I

$$\frac{dV}{dI} = \frac{1}{Z_T} \cdot \frac{d\overline{X}(1 - \tau)}{dI} - \frac{dZ_T}{dI} \cdot \frac{\overline{X}(1 - \tau)}{Z_T^2} \qquad (2.26)$$

Assuming that the characteristics of the firm are unchanged by the additional investment, then $dZ_T = 0$, and imposing the requirement that $dV/dI = 1$ results in

$$\frac{d\overline{X}(1 - \tau)}{dI} \equiv C_T = Z_T \qquad (2.27)$$

Ultimate Flow Model

If cash flows are defined as the ultimate flow to investors (see equation (2.1)), formula (2.6) states the value of the firm. Expanding equation (2.6) and replacing Z with Z_{UF} to indicate the ultimate flow model gives

$$V = \frac{\overline{X}(1 - \tau)}{Z_{UF}} + \frac{\tau i D}{Z_{UF}} \qquad (2.6a)$$

Differentiating equation (2.6a) with respect to I yields

$$\frac{dV}{dI} = \frac{1}{Z_{UF}} \cdot \frac{d\overline{X}(1 - \tau)}{dI} + \frac{1}{Z_{UF}} \cdot \tau i \frac{dD}{dI} - \frac{dZ_{UF}}{dI} \cdot \frac{\overline{X}(1 - \tau) + \tau iD}{Z_{UF}^2} \tag{2.28}$$

Assuming that $dZ_{UF}/dI = 0$, and setting $dV/dI = 1$, results in a marginal cost of capital equal to

$$\frac{d\overline{X}(1 - \tau)}{dI} \equiv C_{UF} = Z_{UF} - i\tau\frac{dD}{dI} \tag{2.29}$$

If future investments are to be financed with the existing proportion of debt, then D/V replaces dD/dI and (2.29) becomes

$$C_{UF} = Z_{UF} - i\tau\frac{D}{V} \tag{2.30}$$

Conditional equivalence between C_T and C_{UF}. So long as the two income streams described above are perpetuities, the marginal costs of capital C_T and C_{UF} must be equal. As proof, subtract C_{UF} from C_T

$$C_T - C_{UF} = Z_T - (Z_{UF} - i\tau\frac{D}{V}) \tag{2.31}$$

Substituting equation (2.16) for Z_T gives

$$C_T - C_{UF} = Z_{UF} - Z_{UF} - i \cdot \tau\frac{D}{V} + i \cdot \tau\frac{D}{V} \tag{2.32}$$

and

$$C_T = C_{UF}{}^{5} \tag{2.33}$$

---

[5] For a discussion of cases in which C_T does not equal C_{UF}, see Arditti and Levy [7].

Modigliani and Miller Model

The marginal cost of capital for the levered firm in a world of taxes is found by differentiating equation (2.20) with respect to I

$$\frac{dV}{dI} = \frac{d\bar{X}(1 - \tau)}{dI} \cdot \frac{1}{\rho^{\tau}} + \frac{\tau dD}{dI} \qquad (2.34)$$

Setting (2.34) to 1 gives

$$\frac{d\bar{X}(1 - \tau)}{dI} = \rho^{\tau}(1 - \tau)\frac{dD}{dI}) \qquad (2.35)$$

Once again by assuming capital proportions subsequent to investment are equivalent to the existing mix, $\frac{dD}{dI} = \frac{D}{V} = L$ and equation (2.35) is written as

$$C(L) = \rho^{\tau}(1 - \tau L) \qquad (2.36)$$

The Robustness of Cost of Capital Formulae

A frequent argument against the M & M model is the set of restrictive assumptions associated with it. The irrelevance of financial leverage except for corporate income taxes is one example. Critics of the M & M model, while acknowledging that tax payments are reduced as financial leverage increases, assert that bankruptcy risk is just too important to assume away;[6] thus the corner solution is unacceptable.

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[6] For one view of bankruptcy risk and the M & M model, see Levy and Sarnat [30, ch. 15].

It does not follow, however, that estimates resulting from the M & M model are biased. That is, in spite of any violations of the underlying assumptions, M & M's view of the world may be sufficiently accurate to avoid systematic errors. Furthermore, there is evidence to support the contention that the M & M model is useful in the "real world." Davis and Sparrow [17] surveyed and critiqued the several common stock financial models of the firm (Gordon, M & M, and Sharpe-Lintner) currently used in rate of return regulation. With respect to the usefulness of the M & M model for rate of return hearings, the authors concluded that all three of the models assume either explicitly or implicitly the financial market conditions required in the derivation of the M & M model.

Examining the Assumptions

A mathematical programming formulation of the problem of financial management to derive weighted average cost of capital formulae was developed by Myers [46]. Equations (2.13) and (2.36) are special cases of his general analysis. For each formula, an examination was made of the magnitude of error when several of the assumptions underlying the model were relaxed.

The general formulation of the Myers' model rests upon the normative objective of the firm, maximization of market value. His derivation of the necessary conditions for maximizing the change in market value provide necessary and sufficient conditions for marginal cost of capital formulae. Table 2.1 summarizes those conditions for the traditional and M & M models (see equations (2.13) and (2.36) respectively).

Table 2.1

Necessary and Sufficient Conditions to Derive Cost of Capital Formulae From Myers' Adjusted Present Value Framework

Assumed Condition	Traditional	M & M
1. Dividend policy is irrelevant	x	x
2. Investment projects are perpetuities	x	x
3. Investment projects make a permanent contribution to debt capacity	x	x
4. Acceptance of an investment project does not shift the target debt ratio	x	x
5. Investment projects do not change the firm's risk class	x	x
6. Leverage is irrelevant except for corporate income taxes		x
7. The firm's assets are expected to generate a constant and perpetual earnings stream	x	
8. The firm is already at its target debt ratio	x	

Source: Myers [46].

Although the conditions for both equations are restrictive, the traditional model requires more conditions for its derivation than does the M & M model. In a subsequent section of his paper, Myers relaxes Conditions 2 and 3 and concludes " . . . the original M & M (cost of capital) formula is acceptably accurate if attention is restricted to projects which do not shift the firm's risk class or target debt ratio. The textbook rule is inferior on all counts."[7]

Growth, Debt Capacity, and the M & M Model

For the most part growth models ignore the fact that the acceptance of profitable investment projects increase the debt capacity of the firm. Indeed, users of growth models seem content to simply substitute $C = \rho(1 - \rho L)$ for ρ in the all equity growth expression, then add this adjusted growth component to a levered no-growth model giving[8]

$$V = \frac{\overline{X}(1 - \tau)}{\rho} + \tau D + \alpha \cdot \overline{X}(1 - \tau)\frac{\rho^* - C}{C(1 + C)} T \qquad (2.37)$$

where

α = the level of future investment in the firm as a percent of after-tax earnings

ρ^* = tax-adjusted expected rate of return on total investment

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[7] Myers [46, p. 19].

[8] This section considers only an after-tax analysis of growth. To simplify the notation ρ replaces ρ^{τ} as the after-tax required yield on an unlevered share of stock, and C replaces C(L) as the M & M marginal cost of capital.

T = number of years that opportunities for investment at $\rho^* > Z$ are expected to last

The Valuation Equation for a Levered Firm with Growth Opportunities

The following analysis demonstrates that although the M & M cutoff criterion for a no-growth firm still obtains, equation (2.37) places too much value on the growth opportunities.[9] Following the Miller-Modigliani analysis [38], it is assumed that an unlevered firm expects to invest a constant fraction of after-tax income at time t, i.e., $\bar{I}(t) - \alpha(1 - \tau)\bar{X}(t)$, where $(1 - \tau)\bar{X}(t)$ symbolizes after-tax expected income in period t, and on that investment expects to earn in perpetuity a net of tax rate of return ρ^* that exceeds ρ for $t = 1, \ldots, T$ and equals ρ thereafter. Then

$$(1 - \tau)\bar{X}(t) = \begin{cases} (1 - \tau)\bar{X}(1)(1 + \alpha\rho^*)^{t-1} \\ \qquad\qquad \text{for } t = 1, 2, \ldots, T + 1 \\ (1 - \tau)\bar{X}(1)(1 + \alpha\rho^*)^{T}\ (1 + \alpha\rho)^{t-T-1} \\ \qquad\qquad \text{for } t = T + 2, T + 3, \ldots, \end{cases} \tag{2.38}$$

and the present value of the unlevered firm V_U at $t = 0$ is

$$V_U = \frac{(1 - \tau)X(1)}{\rho} + \sum_{t=1}^{T} \frac{\rho^*\bar{I}(t)}{(1 + \rho)^t} - \frac{\bar{I}(t)}{(1 + \rho)^t} =$$

$$\frac{(1 - \tau)\bar{X}(1)}{\rho} + \left(\frac{\rho^* - \rho}{\rho}\right) \sum_{t=1}^{T} \frac{\alpha(1 - \tau)\bar{X}(1)(1 + \alpha\rho^*)^{t-1}}{(1 + \rho)^t} \tag{2.39}$$

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[9] This section is from Arditti and Pinkerton [8].

It is of some importance to understand that whether or not the firm continues to invest in time periods subsequent to T, the value of the growth component of (2.39) remains unchanged, since expecting to earn ρ on the funds raised adds zero to present market value. In addition, note that ρ is the discount rate applied to the expected values of $I(t) = \alpha(1 - \tau)X(t)$ to determine the present value of spending $I(t)$ in $t = 1, 2, \ldots, T$.

To make the transition to the valuation expression of the levered firm we must account for the addition to the firm's market value due to the tax deductibility of the interest to be paid on debt. If the firm has an amount of debt D at $t = 0$, we know that the annual tax savings, which are equal to τiD and are assumed to be certain, have present values of τD.

We must now consider the increment to market value engendered by that part of the firm's growth financed by future debt--this is the key to deriving the correct valuation formula. Each year from $t = 1$ to $t = T$ the firm invests $I(t) = \alpha(1 - \tau)X(t)$ where $(1 - \tau)X(t)$ is tax-adjusted income realized in period t and finances $\alpha_L > 0$ of that amount by debt.[10] Therefore, new debt financing amounts to $\alpha_L\alpha(1 - \tau)X(t)$ for $t = 1, \ldots, T$. The actual amount of new debt financing is uncertain at $t = 0$ since it depends on the value that $X(t)$ assumes and that remains unknown until time period t arrives. At that time t, when $X(t)$ becomes known, the amount of debt to be raised during t is certain and the present value at t of the new debt's subsequent annual tax savings is

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[10] The value of α_L is left perfectly generally.

$$\frac{\tau i[\alpha_L\alpha(1 - \tau)X(t)]}{i} = \tau\alpha_L\alpha(1 - \tau)X(t) \text{ for } t = 1, \ldots, T \qquad (2.40)$$

Because at $t = 0$ these future present values are uncertain quantities, the certain rate i cannot be applied to them to convert them to present values at $t = 0$. The problem then is to find the correct discount rate to apply to the expected values of the terms of (2.40).

First note that each present value term, $\tau\alpha_L\alpha(1 - \tau)X(t)$, at time t is proportional to the corresponding t term in the $I(t)$ series where the proportionality factor is $\tau\alpha_L$. Consequently, we can view the terms of (2.40) as the time components of an income stream, $B(t)$, where $B(t) = \tau\alpha_L I(t)$ for $t = 1, 2, \ldots, T$. But this immediately tells us that, in the perfect market environment of M & M's model, the present market value of the $B(t)$ stream, V_B, must equal $\tau\alpha_L$ times the value of the $I(t)$ stream, V_I, where V_I is obtained by discounting the $I(t)$ stream's expected values by ρ (see equation (2.39)).[11] Consequently, we obtain for the value of the levered firm's income-growth component, V_{LG}, that

$$V_{LG} = V_{UG} + \tau\alpha_L V_I \qquad (2.41)$$

---

[11] The proof that if $B(t) \doteq \tau\alpha_L I(t)$ then $V_B = \tau\alpha_L V_I$ is, of course, due to M & M [43]. The mode of the proof is to first assume that $V_B < \tau\alpha_L V_I$ and demonstrate that anyone entitled to a fraction $\varepsilon > 0$ of $I(t)$ at a cost of εV_I can generate the identical income stream if he had ownership rights entitling him to a fraction $(\varepsilon/\tau\alpha_L)$ of $B(t)$ at a cost of $(\varepsilon V_B/\tau\alpha_L)$. Since $(\varepsilon V_B)/(\tau\alpha_L)$ is less than εV_I according to the initial assumption of the proof, then $V_B < \tau\alpha_L V_I$ cannot represent an equilibrium situation. An analogous argument is employed to prove that $VB > \tau\alpha_L V_I$ cannot represent an equilibrium. Consequently, only $VB = \tau\alpha_L V_I$ is consistent with equilibrium.

Recognizing from (2.39) that the value of the unlevered firm's income-growth component, V_{UG}, is equal to $(\frac{\rho^*}{\rho} - 1)V_I$ and using this to solve for V_I in terms of V_{UG}, we have upon substitution for V_I in (2.40)

$$V_{LG} = 1 + (\frac{\tau\alpha_L\rho}{\rho^* - \rho})\ V_{UG} \tag{2.42}$$

We are now in possession of the knowledge necessary to complete the valuation formula for the levered-growth firm. Because the value of the growth-income streams of the levered firm (recall one of those streams in a sequence of present values, each representing the tax savings from issuing new debt at time (t)) is equal to $1 + (\frac{\tau\alpha_L\rho}{\rho^* - \rho})$ of the value of the unlevered firm's growth stream value, V_{UG}, then using (2.39) to substitute for V_{UG} in (2.42) we can write for the levered firm's total value

$$V = \frac{(1 - \tau)\overline{X}(1)}{\rho} + \tau D + \left(\frac{1 + \tau\alpha_L\rho}{(\rho^* - \rho)}\right)\left(\frac{\rho^* - \rho}{\rho}\right)\sum_{t=1}^{T}\left[\frac{\alpha(1 - \tau)\overline{X}(1)(1 + \alpha\rho^*)^{t-1}}{(1 + \rho)^t}\right] \tag{2.43}$$

where we drop the L subscript on V and consider it the value of any firm--if the firm is unlevered then D = 0, $\alpha_L = 0$, and (2.43) reduces to (2.39).

Writing the summation in closed form we have

$$V = \frac{(1 - \tau)X(1)}{\rho} + \tau D + \left[\frac{1 + \tau\alpha_L\rho}{(\rho^* --\rho)}\right]\left(\frac{\rho^* - \rho}{\rho}\right)\left[\frac{\alpha(1 - \tau)X(1)\left[1 - \frac{1 + \alpha\rho^*}{1 + \rho}\right]^T}{\rho - \alpha\rho^*}\right] \tag{2.44}$$

or

$$V = \frac{(1 - \tau)\overline{X}(1)}{\rho} + \tau D + \left[\frac{(\rho^* - \rho)}{\rho} + \tau\alpha_L\right]\frac{\alpha(1 - \tau)\overline{X}(1)}{(\rho - \alpha\rho^*)}\left[1 - \left(\frac{1 + \alpha\rho^*}{1 + \rho}\right)^{T}\right] \tag{2.45}$$

To write (2.45) in the simplifying form of (2.37), assume that $h = \frac{1 + \alpha\rho^*}{1 + \rho}$ is close to 1 and expand h^T about 1, dropping all terms but the first two of the Taylor series. This results in

$$V = \frac{(1 - \tau)\overline{X}(1)}{\rho} + \tau D + \left[\frac{(\rho^* - \rho)}{\rho} + \tau\alpha_L\right]\left[\frac{\alpha(1 - \tau)\overline{X}(1)\cdot T}{(1 + \rho)}\right] \tag{2.46}$$

Equation (2.46) provides the correct specification of a levered-growth firm's valuation in M & M's framework.

A glance at equations (2.45) and (2.46) reveals that in the M & M framework a firm may benefit from a new project even if that project is not expected to earn a rate of return in excess of that required for the unlevered firm, ρ. The reason is that if $[(\rho^* - \rho)/\rho]$ were zero or to some extent even negative, the benefits derived by that project increasing the firm's debt capacity--those benefits being represented by the $\tau\alpha_L$ term--will offset the project's unfavorable rate of return. Thus, the appropriate capital budgeting criterion is

$$\frac{(\rho^* - \rho)}{\rho} + \tau\alpha_L > 0 \tag{2.47}$$

or

$$\rho^* > \rho(1 - \tau\alpha_L) \tag{2.48}$$

which is precisely the M & M cutoff criterion for a no-growth firm when (dV/dI) of the perpetuity valuation equation was set equal to one and an expression for $(1 - \tau)(d\bar{X}/dI)$ was subsequently solved for (see [42]).

A Comparison with the M & M Equation

In order to contrast the M & M valuation expression (2.37) with (2.46) at a given capital structure, we must ensure that the parameter values of the two expressions are consistent with this assumed capital structure. We proceed by considering a firm that has an investment growth opportunity at t = 1 and none after that. Let the size of the expected investment necessary to take advantage of this opportunity be $\bar{I}(1)$ and symbolize the market value of the firm in the absence of growth by V_{LNG} where

$$V_{LNG} = \frac{(1 - \tau)\bar{X}(1)}{\rho} + \tau D \qquad (2.49)$$

The debt-value ratio that corresponds to this no-growth segment of the firm's business is

$$\frac{D}{V_{LNG}} = \frac{D}{\frac{(1 - \tau)\bar{X}(1)}{\rho} + \tau D} \qquad (2.50)$$

Assume that the firm wishes to finance its growth opportunity at the same debt-value ratio that it applies to its no-growth business; therefore, the firm's overall debt-value ratio is expected to always equal D/V_{LNG}. This requirement is met if the expected amount of debt used to finance the growth opportunity, $\alpha_L\bar{I}(1)$, divided by the sum of the expected investment cost, $\bar{I}(1)$, and the surplus value generated by the investment at t = 1, $V_{LG}(1)$, equals D/V_{LNG}.[12] That is

$$\frac{\alpha_L \bar{I}(1)}{\bar{I}(1) + V_{LG}(1)} = \frac{D}{V_{LNG}} \tag{2.51}$$

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[12] Since investment takes place at t = 1, the firm should really be expected to use the debt-value ratio of its no-growth business at t = 1. However, assuming that the firm has the same amount of debt outstanding at t = 1 as it does at t = 0, the relevant debt-value ratio $D/V_{LNG}(1)$, where $V_{LNG}(1)$ represents the value of the levered no-growth income stream at t = 1, is exactly equal to the debt-value ratio at t = 0, D/V_{LNG}. To see this note that V_{LNG}, the no-growth income stream's value at t = 0, is given by

$$\text{(i)} \quad V_{LNG} = \frac{(1 - \tau)\bar{X}(1) + V_{LNG}(1)}{1 + C}$$

or

$$\text{(ii)} \quad V_{LNG}(1) = (1 + C)V_{LNG} - (1 - \tau)\bar{X}(1)$$

However, because

$$\text{(iii)} \quad V_{LNG} = \frac{(1 - \tau)\bar{X}(1)}{\rho} + \tau D$$

Then

$$\text{(iv)} \quad (1 - \tau)\bar{X}(1) = \rho(1 - \frac{\tau D}{V_{LNG}})V_{LNG} = CV_{LNG}$$

Substituting expression (iv) into (ii) gives

$$\text{(v)} \quad V_{LNG}(1) = (1 + C)V_{LNG} - CV_{LNG} = V_{LNG}$$

Therefore,

$$\text{(vi)} \quad \frac{D}{V_{LNG}(1)} = \frac{D}{V_{LNG}}$$

But (2.51) tells us that the appropriate α_L value is given by

$$\alpha_L = \frac{D}{V_{LNG}} \cdot \frac{\bar{I}(1) + V_{LG}(1)}{\bar{I}(1)} \tag{2.52}$$

Because (2.42) provides a general relationship between the value of the levered growth component and the unlevered growth component starting at any point in time, we have that

$$V_{LG}(1) = 1 + \frac{\tau\alpha_L\rho}{\rho^* - \rho} V_{UG}(1) \tag{2.53}$$

where $V_{UG}(1)$ represents the value of the unlevered growth component at t = 1. However,

$$V_{UG} = \frac{V_{UG}(1)}{(1 + \rho)} \tag{2.54}$$

because both the outflows and inflows attributed to growth do not begin until t = 1. Using (2.54) to eliminate $V_{UG}(1)$ in (2.53), one obtains

$$V_{LG}(1) = \left[1 + \frac{\tau\alpha_L\rho}{\rho^* - \rho}\right] V_{UG}(1 + \rho) \tag{2.55}$$

But by (2.39)

$$V_{UG} = \left(\frac{\rho^* - \rho}{\rho}\right)\frac{\bar{I}(1)}{(1 + \rho)} \tag{2.56}$$

Therefore, (2.55) can be written as

$$V_{LG}(1) = \left[1 + \frac{\tau\alpha_L\rho}{\rho^* - \rho}\right]\left(\frac{\rho^* - \rho}{\rho}\right)\bar{I}(1) \tag{2.57}$$

Combining (2.52) and (2.57) we have

$$\alpha_L = \frac{D}{V_{LNG}}\left[1 + \left(\frac{\rho^* - \rho}{\rho} + \tau\alpha_L\right)\right] \tag{2.58}$$

Expression (2.58) shows that if a firm has a growth opportunity at $t = 1$, i.e., $\rho^* > \rho(1 - \tau\alpha_L)$ or $[(\rho^* - \rho)/\rho + \tau\alpha_L] > 0$, then $\alpha_L > D/V_{LNG}$. This result is sensible because if the investment generated surplus equity value of $V_{LG}(1)$ then it must be expected that enough debt will be used in financing $\overline{I}(1)$ so that $V_{LG}(1)$ is covered by the appropriate debt-equity ratio.

An equivalent and useful expression for the appropriate α_L value can be obtained by isolating α_L in expression (2.58). This yields

$$\alpha_L = \frac{D}{V_{LNG}}\left[\frac{\rho^*}{\rho(1 - \tau D/V_{LNG})}\right] \tag{2.59}$$

Recognizing $\rho(1 - \tau D/V_{LNG})$ as M & M's C gives

$$\alpha_L = \frac{D}{V_{LNG}}\left(\frac{\rho^*}{C}\right) \tag{2.60}$$

Thus, if $\rho^* > C$, then $\alpha_L > D/V_{LNG}$. The two criteria, (2.58) and (2.60), that describe the relative values of α_L and D/V_{LNG} are consistent.

We are now prepared to compare the M & M expression of V_{LG} (see (2.37)), which is

$$\left(\frac{\rho^* - C}{C}\right)\frac{\overline{I}(1)}{(1 + C)}, \tag{2.61}$$

with the V_{LG} expression derived in this paper (see (2.46)) and given by

$$\left[\frac{\rho^* - \rho}{\rho} + \tau\alpha_L\right]\frac{\overline{I}(1)}{(1 + \rho)}\ . \tag{2.62}$$

Employing (2.60) to replace α_L in (2.62) alters (2.62) to

$$\left[\frac{\rho^* - \rho}{\rho} + \tau\left(\frac{\rho^*}{C}\right)\frac{D}{V_{LNG}}\right]\frac{\bar{I}(1)}{(1 + \rho)}$$

$$= \frac{(\rho^* - \rho)\ (1 - \tau D/V_{LNG}) + \tau\rho^* D/V_{LNG}}{C}\ \frac{\bar{I}(1)}{(1 + \rho)} \tag{2.63}$$

or equivalently

$$\left(\frac{\rho^* - C}{C}\right)\left(\frac{\bar{I}(1)}{(1 + \rho)}\right) \tag{2.64}$$

Since $\rho > C = \rho(1 - \tau D/V_{LNG})$ when $D/V_{LNG} > 0$, a comparison of (2.61) with (2.64) reveals that

$$\left(\frac{\rho^* - C}{C}\right)\frac{\bar{I}(1)}{(1 + C)} > \left(\frac{\rho^* - C}{C}\right)\frac{\bar{I}(1)}{(1 + \rho)} \tag{2.65}$$

Consequently, the M & M valuation expression places too high a value on growth opportunities.

The Assumption of Equal Business Risk

In their classic analysis of the effect of debt financing on security prices Modigliani and Miller [41] found it necessary to classify firms into homogeneous groups that were perfect substitutes for one another. That assumption was also found to be of principal importance to the cost of capital models reviewed above.

It is convenient to partition the total uncertainty surrounding the expected earnings of a firm into two parts: uncertainty arising from economic, social, and political factors and uncertainty resulting from the type of financing used by the firm. The former is called business risk when associated with a probability distribution of investors' assessment of a firm's earnings and investment. Parameters of the distribution are influenced by demand elasticity, firm size, asset structure, cost structure, labor relations, control of raw materials, geographic region, degree of regulation, and more.

There is evidence refuting the general contention that equivalent risk classes are bounded by industrial classification.[13] But electric utilities, it has been argued, are unique. Many consider them to be the best example of an industry homogeneous in business risk (cf. [26], [40], and [55]). In addition to similarities in technology and cost structure, the electric utility industry is characterized by firms with regulated rates of return and relatively inelastic demand curves. However, a close examination does reveal important differences. First, the regulating process is administered by autonomous state commissions. Second, during the past two decades, technological dissimilarities have become more pronounced as nuclear generating plants have been constructed. Also, fuel sources have given rise to dissimilar cost structures among firms.[14] Thus, in spite of any broad generalities, if one is to seriously use electric utilities to satisfy the equivalent risk class assumption the appropriateness of the sample should be confirmed.

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[13] See Wippern [56], Gonedes [23], and the discussion of each in Chapter III.

[14] For example, electric utilities in New England rely more heavily on expensive foreign oil than do utilities in Texas. In addition to the foreign-domestic mix of fuel, the type of fuel (natural gas, oil, coal) also plays a critical role in determining total costs. On the other hand, government pricing schemes tend to homogenize these fuel cost differences over all firms.

Defining Equivalent Risk Classes

Risk is an ex ante concept. Nevertheless, one can make the assumption that future risk is similar to past risk and use an objective measure of past risk as an estimate of future risk. The variability about an average is frequently taken as a sufficient statistic of risk. However, the relationship between variability and risk is elusive. For example, earnings' variability which is fully anticipated is riskless. Furthermore, because firms are able to disguise earnings' variability to some extent through various accounting procedures, identification of true earnings' variability is difficult. Finally, upward variability in earnings is not viewed with the disdain reserved for downward variability.

Modigliani and Miller condition.

An early rigorous definition of business risk is by Modigliani and Miller. They assumed firms could be classified

> . . . such that the return on the shares issued by any given class is proportional to (and hence perfectly correlated with) the return on the shares issued by any other firm in the same class. Firms within a particular class differ only by a scale factor, and after adjusting for scale (dividing return by expected return), any two firms i and j would be in a risk class if the respective distributions of their ratios of return to expected return were identical.[15]

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[15] Modigliani and Miller [41, p. 267].

Specifically, M & M defined risk classes as follows. Let

$\tilde{X}_i$ = the average value of EBIT generated by the assets of firm i over time. $\tilde{X}_i$ is a random variable, determined by the stream of earnings over time, $X_i(1)$, $X_i(2)$, . . . , $X_i(T)$, subject to the joint probability distribution $\chi_i[X_i(1), X_i(2), \ldots, X_i(t)]$, and defined as $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} X_i(t)$ with probability distribution $\Phi(X_i)$.

$\overline{X}_i$ = $E(X_i)$ = expected value of X_i determined by the random variable $\tilde{X}_i$ with a probability distribution $\Phi(\tilde{X}_i)$ and defined as $\int X_i \Phi(X_i) dX_i$.

Then the ratio $\dot{X}_i = \tilde{X}_i/\overline{X}_i$ has some probability distribution $\psi_i(X)$ and firms i and j will be in the same risk class if and only if

$$\psi_i(\dot{X}) = \psi_j(\dot{X}) \qquad (2.66)$$

While pointing out that a class of securities identified by (2.66) is not "identical to that of the industry,"[16] M & M nevertheless believed that for empirical work industrial classification adequately approximated this condition. They used samples of firms belonging to the oil and electric utility industries in their initial empirical tests, considering each industry as a homogeneous risk class.

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[16] Modigliani and Miller [41, fn. 9].

There was and still is substantial disagreement over the generality of M & M's use of industry classification as a device for partitioning firms into equivalent risk classes. For example, Weston [55] criticized M & M's selection of the oil industry as a homogeneous risk class.

> The difference between firms in the oil industry, in particular, are great. For example, in a group of 42 oil companies used by Modigliani and Miller would be found the following diversity: fully integrated oil companies, oil companies strong in refining, oil companies strong in distribution, some regional in their operations, some with heavy investments in troubled international regions; some with stable, assured or rising income from petrochemicals or uranium or other minerals.[17]

He found it necessary for his empirical tests of M & M's propositions to use the electric utility industry. M & M also used electric utilities in their major empirical cost of capital study [39] because they considered utilities less heterogeneous than other industry groups. In another major study of M & M propositions, Alexander Barges [10] selected three samples with differing characteristics: Class 1 railroads, department store companies, and cement companies. While admitting that much of the data for intra-industry firms was to some extent heterogeneous, e.g., size, market served, stability, and earning power, Barges claimed that the heterogeneity was not so large as to lead to systematic biases. Finally, in [27], Haley and Schall note:

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[17] Weston [55, p. 108].

> Even with perfect markets, the M & M assumption that firms can be placed into risk classes with returns that have identical distributions (except for a possible scale factor) and are perfectly correlated is of questionable validity. No such risk classes may exist, or, if they do exist, a risk class may include streams with differing distributions but equivalent risk.[18]

Fama and Miller condition. Professors Fama and Miller [22, p. 161] have provided a more restrictive equivalent risk class condition. Firms i and j are in the same risk class if for all periods

$$X_i(t) = \lambda_i X_j(t) \tag{2.67a}$$

and

$$I_i(t) = \lambda_i I_j(t) \tag{2.67b}$$

where

λ_i = a proportionality factor, the same over all t for both earnings and investments.

The authors also point out that even in periods before t when earnings and investments are uncertain, ". . . investors must agree that whatever values earnings and investment outlays take in any period, for these two firms they are always proportional by the factor λ_i and hence perfectly correlated."[19]

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[18] Haley and Schall [27, p. 228].

[19] Fama and Miller [22, p. 161].

Comparison of the two risk class definitions. Exactly what are the differences between the two definitions of an equivalent risk class? First, neither in their early empirical work [41] nor in their later more extensive study [39] did M & M impose condition (2.67b) on investment outlays. However, if firms are to be true substitutes for one another, some such condition is required to ensure that income generated per dollar of invested assets is identical. An obvious difference then between the Modigliani and Miller and the Fama and Miller definitions of equivalent risk classes is the condition of proportional investment outlay.

An additional contrast exists because income condition (2.67a) requires all elements of the future uncertain income stream $X(t)$ to differ by only λ_i when realized. This assures that the distributions of scaled earnings are identical and therefore perfectly, positively correlated. On the other hand, condition (2.66) requires only that the two distributions be equal, not that corresponding elements of the income streams associated with those distributions are identical. The difference between the two earnings stream conditions then boils down to this: Condition (2.66) imposes fewer restrictions on the characteristics of the earnings stream, but necessarily imposes greater restrictions upon the preferences of investors. Investors need be concerned only with average earnings over time, not with what realized earnings will be each period as required by condition (2.67a).

Summary

Three partial equilibrium cost of capital specifications have been reviewed for the non-growth firm: traditional, ultimate flow, and M & M. Also, marginal cost of capital formulae were derived for each specification. It was shown that the marginal costs of capital associated with the traditional and ultimate flow models were identical under conditions of perpetual and constant cash flow. Arditti [4] and Arditti and Levy [7] have shown that when other conditions exist, the traditional model incorrectly specifies the firm's marginal cost of capital. Necessary and sufficient conditions to derive the marginal cost of capital were briefly reviewed. Myers [46] has shown that the M & M model actually requires fewer assumptions than does the traditional model. Furthermore, when two assumptions common to both models were relaxed, the M & M model proved to be more robust than the traditional model.

Alteration of the perpetual earnings valuation model to allow for growth was introduced by way of a criticism of the M & M growth model for a levered firm. Arditti and Pinkerton [8] have shown that the M & M valuation model (see equation (2.37)) places too high a value on the growth term.

The final part of the chapter reviewed the theoretical requirements of the equivalent risk class assumption (a necessary and sufficient condition for deriving marginal cost of capital formulae).

CHAPTER III

REVIEW OF EMPIRICAL COST OF CAPITAL STUDIES

Because a single chapter cannot review in depth all of the important contributions regarding the estimation of the cost of capital, an alternative approach is needed. In the next section therefore, several significant studies will be summarized. They are: Modigliani and Miller [41], Barges [10], Weston [55], Wippern [57], and Brigham and Gordon [13]. Then in subsequent sections of the chapter a detailed review will be conducted of Miller and Modigliani [39], Gordon [26], Litzenberger and Rao [32], and McDonald [36]. The articles selected for the more extended review are not necessarily better than the studies placed in the preceding resume or omitted. Rather, the selection is based on a desire to demonstrate methodology as well as results. The Gordon monograph and the Litzenberger and Rao article, for example, made use of ordinary least-squares regression; Miller and Modigliani employed two-stage least-squares regression; and McDonald's use of iterative least-squares regression enabled him to explicitly include the nonlinearity of the growth term in his statistical model. Also, some of the empirics were duplicative of other work and therefore were only summarized. An example is the Gordon monograph, an updated, more exhaustive version of Brigham and Gordon.

The final section of the chapter again addresses the assumption of equivalent business risk among firms, this time by reviewing two empirical studies. The first one is by Wippern [56] who used classical analysis of variance and multiple comparison tests to examine business risk among industries. The second study is by Gonedes [23]. He rejected the use of the parametric model in Wippern's analysis, employing instead nonparametric analysis of variance to reject the hypothesis of equal business risk among and within industries.

Several Cost of Capital Studies

Modigliani and Miller

In Chapter II the M & M defined relationship between required rate of return on equity capital and financial leverage in a world of no taxes was given as

$$\frac{\overline{\pi}}{S} = \rho + (\rho - i)\frac{D}{S} \tag{2.19}$$

When taxes were included in the analysis, the after-tax required rate of return on equity was stated as

$$\frac{\overline{\pi}^{\tau}}{S} = \rho^{\tau} + (1 - \overset{\tau}{\rho})(\rho^{\tau} - i)\frac{D}{S} \tag{2.25}$$

Statistical tests of the above equations have generally been based on a cross section of firms, presumably of the same class of business risk. As noted in Chapter II, unlevered firms within a given risk class must have identical rate of return distributions (see condition (2.66)).

A statistical equation can be formed for equations (2.~~20~~19) and (2.25) as

$$k = a_o + a_1 \frac{D}{S} + u \qquad (3.1)$$

where a_o and a_1 are regression coefficients with expected values equal to either ρ and $(\rho - i)$ respectively for equation (2.~~20~~19) or ρ^τ and $(1 - \tau)(\rho^\tau - i)$ respectively for equation (2.25). In either case k is a linear function of the debt-to-equity ratio. However, if k is a curvilinear function of financial leverage then

$$k = a_o + a_1 \frac{D}{S} + a_2 (\frac{D}{S})^2 + u \qquad (3.2)$$

may be a more appropriate statistical equation for testing the relationship hypothesized in equations (2.~~20~~19) and (2.25). In their original paper Modigliani and Miller [41] used data gathered from earlier studies to substantiate their several propositions on the cost of capital. The results of their regressions tended to support their Proposition II. That is, estimates of a_o and a_1 were statistically significant and of "plausible" magnitudes. The estimate for a_2 in equation (3.2) was either not significant or it had the wrong sign. M & M measured k as the inverse of the firm's price-to-earnings ratio, and D/S as the market value of debt divided by the market value of equity.

Several Criticisms

A number of problems with M & M's early empirical work have been pointed out. Barges [10] noted the existence of spurious correlation because the market value of equity, appearing on both sides of the statistical equation, biased the results toward being significant. For the most part, Barges replicated M & M's tests with his own samples of homogeneous firms but measured the debt-to-equity ratio with historical (or book) values. His results were consistent with the traditional view of a saucer shaped average cost of capital function. Evidence supporting the traditional view was also gathered by Wippern [57]. He replicated M & M's tests, but used a ratio of the cash-flow cost of the debt to earnings before interest and taxes as an indication of financial leverage. Another criticism of the original M & M empirical work was made by Weston [55]. He pointed out that the earnings-to-price ratio M & M used to measure k was appropriate only for firms with no anticipation of further growth in earnings. Thus, M & M's use of this ratio as a measure of the required rate of return on equity biased their results. By adding additional variables to the right-hand side of the statistical equation, specifically total assets and the growth rate in earnings per share, Weston's results also supported the traditional view.

Brigham and Gordon

Professors Brigham and Gordon [13] based their analysis on the stock valuation equation

$$P = \int_{0}^{\infty} \overline{d}(t)e^{-k \cdot t}dt \qquad (3.3)$$

where

P = price per share of common stock

$\overline{d}(t)$ = expected dividend per share in period t

Assuming that the firm retains an expected fraction α of the current year's after-tax earnings and earns an after-tax rate of return $\rho*$ on this incremental equity, then the expected dividend in any future period t will be

$$\bar{d}(t) = (1 - \alpha)(1 - \tau)x(o)e^{\alpha\rho* t} \tag{3.4}$$

where

$(1 - \tau)x(o)$ is defined as net income available to each share of common stock at $t = 0$.

Substituting (3.4) into (3.3) and integrating the latter, assuming $\alpha\rho* < k$, gives

$$P = \frac{(1 - \alpha)(1 - \tau)\bar{x}(1)}{k - \alpha\rho*} \tag{3.5}$$

Recognizing $(1 - \alpha)(1 - \tau)\bar{x}(1)$ as $\bar{d}(1)$, equation (3.5) was solved as

$$\frac{\bar{d}(1)}{P} = k - b\rho* \tag{3.6}$$

If all earnings are paid in dividends, then M & M's required return on equity for a public utility is written as equation (2.20) above, which substituted into (3.6) gives[1]

$$\frac{\bar{d}(1)}{P} = \rho - \alpha\rho* + (\rho - i)\frac{D}{S} \tag{3.7}$$

or in statistical form

$$\frac{\bar{d}(1)}{P} = a_o + a_1\alpha\rho* + a_2\frac{D}{S} + u \tag{3.8}$$

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[1] The fact that Brigham and Gordon used the required rate of return formula for a no-tax world is due to the after-tax regulation of utilities. More is said on this point in the next section.

where

(1) $E(a_0) = \rho$, $E(a_1) = -1$, and $E(a_2) = (\rho - i)$ if there is no impact on share price from either dividend policy or financial leverage, or

(2) $-1 < E(a_1) < 0$ indicates that k is an increasing function of $\alpha\rho^*$, and

(3) $E(a_2) < (\rho - i)$ indicates investors' preference for corporate rather than personal leverage.

In their statistical formulation of equation (3.7) the authors included some additional variables to account for the heterogeneity of the sample. Their results, the authors concluded, were generally consistent with the traditional theory of capital structure. Brigham and Gordon also formulated a nonlinear model which is nearly identical to Gordon's nonlinear model discussed below.

Miller and Modigliani: Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954-57

Miller and Modigliani [39] employed two-stage regression to calculate estimates of the cost of capital to the electric utility industry for the years 1954, 1956, and 1957. They found no evidence of sizable leverage or dividend effects for the utility industry of the type assumed in traditional finance theory. Furthermore, estimates of the cost of equity capital and average cost of capital calculated by their model followed, they claimed, reasonably well the existing rates on long-term bonds over the period of study.

The Miller and Modigliani Study

The M & M valuation model for a levered firm with growth potential is

$$V = \frac{\bar{X}(1 - \tau)}{\rho} + \tau D + \alpha\bar{X}(1 - \tau)\frac{\rho^* - C}{C(1 + C)} T \qquad (2.37)$$

From (2.41), the following statistical equation was formed

$$\frac{V - \tau D}{A} = a_o \frac{1}{A} + a_1 \frac{\bar{X}(1 - \tau)}{A} + a_2 \frac{\overline{\Delta A}}{A} + u \qquad (3.9)$$

where

a_o = a measure of the effect of scale on valuation

$a_1 = 1/\rho$

a_2 = the contribution of growth to value equal to $\frac{\rho^* - C}{C(1 + C)} T$

A = total assets at book value

$\overline{\Delta A}$ = an estimate of $\bar{X}(1 - \tau)$ taken as the average change in assets over a four year period

u = a random disturbance term

The coefficient a_1 was of principal importance to M & M because its inverse, the after-tax cost of capital for a pure equity firm, was employed in calculating the marginal cost of capital function

$$C(L) = \rho^{\tau}(1 - \tau L) \qquad (2.36)$$

To avoid errors in published accounting data, M & M adopted an instrumental variable for expected long-run tax-adjusted earnings. To implement this approach an estimate of $(\bar{X}^{\tau} - \tau\bar{R})/A$ was first calculated as

$$\frac{(\bar{X}^{\tau} - \tau\overline{Int})}{A} = b_o + b_1 \frac{1}{A} + b_2 \frac{\overline{\Delta A}}{A} + b_3 \frac{D}{A} + b_4 \frac{PF}{A} + b_5 \frac{Div}{A} + v \qquad (3.10)$$

where newly defined variables are

$\overline{Int}$ = average interest payments

PF = market value of firm's preferred stock

Div = total dividends paid to common stock

v = a random disturbance term

In the second stage of the regression the fitted values of (3.10) were used for earnings in (3.9) to calculate the estimates $\hat{a}_o$, $\hat{a}_1$, $\hat{a}_2$ reported in Table 3.1. The regressions for each year were fitted with and without a constant term in order to substantiate M & M's hypothesis that the true constant term of the statistical equation was zero; i.e., the constant term not being significantly different from zero served, the authors believed, to confirm their use of the instrumental variable approach. Coefficients associated with dividend payout and total debt, additional explanatory variables added to the second-stage regression, were found to be not significantly different from zero.

Table 3.2 reports M & M's estimates of the cost of equity capital for a pure equity stream, the average earnings yield on shares, and the reciprocal of the market-to-book value ratio. The estimated cost of equity capital was calculated as the reciprocal of coefficient $\hat{a}_1$ (see equation (3.9)). The relative importance of each of the variables in equation (3.9)--cost of equity, growth, leverage (debt and preferred), and size--is reported in Table 3.3.

Critique of the Miller and Modigliani Study

Criticism of M & M's empirical study has generally fallen into one of the following categories: (1) an upward biased estimation of $\hat{a}_1$; (2) the inadequacy of the designed tests to measure the impact of financial leverage and dividend policy on firm value; and (3) heterogeneous results when cross-sectional estimates were calculated for additional years.

Table 3.1

Second-Stage Estimates with Computed Earnings
Dependent Variable: $(V-\tau D)/A$

Year	Constant	Computed Earnings $(\overline{X}^{\tau}-\tau\overline{R})/A$	Size $1/A\cdot 10^7$	Growth $\overline{\Delta A}/A$	Multi. R
1957	-.004 (.08)	16.2 (1.7)	-.280 (.08)	1.37 (.24)	.87
1956	.054 (.08)	15.6 (1.6)	-.122 (.07)	.847 (.23)	.87
1954	.072 (.10)	18.1 (2.2)	-.234 (.07)	.286 (.17)	.77
1957	[a]	16.1 (.46)	-.280 (.08)	1.36 (.23)	.88
1956	[a]	16.7 (.40)	-.114 (.07)	.896 (.21)	.87
1954	[a]	19.7 (.45)	-.244 (.07)	.299 (.18)	.73

Source: Miller and Modigliani [39].

[a] Constrained to equal zero.

Table 3.2

Estimated Cost of Equity Capital and Some Alternative Measure of Equity Costs

Year	Estimated Cost of Equity Capital (ρ)		Average Earnings Yield on Shares ($\bar{\pi}^{\tau}/S$)		Reciprocal of Price to Book-Value Ratio (E/S)	
	Amt.	As per-cent of 1954	Amt.	As per-cent of 1954	Amt.	As per-cent of 1954
1957	.062	122	.070	106	.64	105
1956	.060	118	.070	106	.63	103
1954	.051	100	.066	100	.61	100

Source: Miller and Modigliani [39].

Table 3.3

Relation Between Average Earnings Yield and the Cost of Equity Capital

Component	1957		1956		1954	
Average Earnings Yield		.070		.070		.066
Less:						
Leverage Effect		.019		.017		.014
Debt	.016		.014		.012	
Preferred Stock	.003		.033		.002	
Size Effect		.002		.001		.002
Plus:						
Growth Effect		.014		.008		.003
Total		.063		.060		.053
Estimated Cost of Equity Capital		.062		.060		.051

Source: Miller and Modigliani [39].

Crockett and Friend. For the most part Crockett and Friend [16] questioned the existence of biases in the statistical model. First, they argued, size must be explicitly taken into account. Through a diversification effect, the cost of equity capital could vary with size thus creating a nonlinear relationship between $(V - \tau D)$ and $(\overline{X}^{\tau} - \tau\overline{R})$. Enforced linearity of that relationship would result in $\hat{a}_1$ being biased upward. A second potentially upward bias in $\hat{a}_1$ is caused by the replacement of actual earnings with instrumental variables. That is, the inclusion of disbursements such as dividends, debt service, and preferred stock dividends could have independent effects upon value. Finally, the selection of the average change in assets as the growth variable could bias $\hat{a}_1$ upward. Crockett and Friend, while in general agreement with M & M about the components of growth, believed that the rate of return on existing assets $((\overline{X}^{\tau} - \tau\overline{R})/A)$ and the rate of return anticipated on additional assets were highly positively correlated. Thus, impacting the latter into the $\hat{a}_2$ coefficient would create an upward bias in $\hat{a}_1$ due to the positive correlation between $\hat{a}_1$ and $\hat{a}_2$. Gordon [24] also came to this conclusion.

M & M [40] responded to each of Crockett and Friend's points. First, plots of $V - \tau D$ on $\overline{X}^{\tau} - \tau\overline{R}$ showed no evidence of the hypothesized curvature. Second, a specific size variable (log of total assets) was initially included to test for serious distortion; none was found. Third, dividends were excluded from the first-stage regression and almost identical results obtained. Finally, with respect to the growth term, M & M countered that the argument presented by Crockett and Friend was inconsistent. Rate of return on additional investment must exceed the cost of capital for growth to exist at all. Thus, it is inconsistent to argue that the growth coefficient is underestimated while the existing earnings coefficient is overestimated. Furthermore, M & M concluded, numbers obtained by Crockett and Friend for comparison to theirs were extremely close when similar weights (M & M used market value and Crockett and Friend used book value) and marginal tax rates ($\tau = .52$ according to M & M compared to $\tau = .50$ for Crockett and Friend) were used.

Gordon. Myron Gordon's [24] criticism was directed primarily at M & M's test for the significance of leverage and dividend policy as determinants of firm value. Gordon contended that since the sample was drawn exclusively from the electric utility industry, the particular characteristics of rate of return regulation must be accounted for in the model. Specifically, since utilities are regulated by their after-tax rate of return, interest payments should have been treated as an after-tax expense. Because the tax subsidy is passed directly on to consumers through lower total revenue requirements, Gordon believed that the τD benefit did not exist for utilities. Therefore, instead of equation (3.9), M & M should have used

$$\frac{V}{A} = a_o + a_1 \frac{\overline{X}^{\tau}}{A} + a_2 \frac{1}{A} + a_3 \frac{\overline{\Delta A}}{A} + u \qquad (3.11)$$

Furthermore, if debt and preferred stock variables had been included in equation (3.11) rather than (3.9), each of the coefficients would have been positive and significant.

With respect to the significance of dividend policy, Gordon argued that M & M's statistical equation precluded testing for two reasons. One, the dependent variable should have been the value of common shares rather than total firm value, and two, M & M's dividend variable measured the average fraction of earnings on all assets paid in dividends rather than a payout ratio of dividends relative to earnings available to common stock. Since dividend policy should be highly correlated with share value, M & M's variable could be expected to display lower correlation with $V - \tau D$ than with share value.

M & M [40] conceded that their use of electric utilities for the test sample brought into question the independence of $X/\overline{X}$ and leverage, as opposed to Gordon's view that $X^{\tau}/\overline{X}^{\tau}$ is independent of leverage. But, they argued, if $X^{\tau}/\overline{X}^{\tau}$ is independent of leverage, then it follows that the tax subsidy to consumers causes $X/\overline{X}$ to be a decreasing function of leverage.[2] Furthermore, if investors have a preference for levered firms for reasons other than the tax savings, the coefficient for preferred stock in the second-stage regression should have been positive when in fact it was not significant. As far as any downward bias in the dividend coefficient, M & M admitted to some bias but dismissed it as being so small as not to alter their major conclusion.

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[2] For additional discussion relating to the impact of financial leverage on regulated industries, see Elton and Gruber [20] and [21] and Gordon and McCallum [25].

Robichek, McDonald, and Higgins. A replication of M & M's methodology for additional years was conducted by Robichek, McDonald, and Higgins [49]. They noted that the coefficients changed dramatically. For example, the increased value of the earnings coefficient through 1964 resulted in a decline of some 25 percent in the cost of equity estimate. However, M & M [40] pointed out that during this same time the average market value for securities rose 43 percent, while earnings increased only a 14 percent. M & M also countered that the two sets of results were not really comparable. They had recorded debt and preferred stock at market value, while their critics had used book value. Also, deferred taxes became increasingly important in the late fifties and early sixties.

Summary of the Miller and Modigliani Study

The M & M study is important for several reasons. One is the two-stage least-squares regression technique employed, the first employed in financial modeling. Also, it provided estimates of the average cost of capital, which although low cannot be ignored. The weakest portion of their analysis appears to be the leverage and dividend tests which even M & M admitted did not close the issue.

Gordon: The Cost of Capital to a Public Utility

In his detailed monograph Myron J. Gordon [26] refined and extended his and Eugene F. Brigham's [13] earlier work. The perfectly competitive capital markets cost of capital (hereafter PCCM) is defined by Gordon as (see equation (2.15))

$$Z = \frac{k \cdot S + 1 \cdot D}{S + D} \tag{3.12}$$

As pointed out above, the principal difference between those who see a world described by M & M and those who believe an advantage exists for moderate amounts of debt in a firm's capital structure rests upon the cost of equity capital

$$k = \rho + \gamma\frac{D}{S} \qquad (3.13)$$

If $\gamma = (\rho - i)$, then $Z = \rho$ and the cost of capital is independent of financial leverage. On the other hand, if $\gamma < (\rho - i)$, Z is a decreasing function of financial leverage up to some level of debt. Thereafter interest rates increase due to the increased possibility of default. For a public utility Gordon believes that except for k historical rather than market values should be used in (3.12), giving

$$Z = \frac{k \cdot E + e \cdot B}{E + B} \qquad (3.14)$$

with

$$k = \rho + \gamma\frac{B}{E} \qquad (3.15)$$

A major portion of Gordon's empirical work was devoted to the testing of equations (3.13) and (3.15). In the process the M & M and traditional dividend rate and stock financing theorems were also examined. Gordon concluded that the validity of all of them is questionable with respect to public utilities. His alternative nonlinear model is also presented and empirically tested.

Test of PCCM Leverage Theorem

For equation (3.13) Gordon estimated coefficients for the statistical equation

$$k = a_o + a_1 \frac{D}{S} + a_2 g + a_3 se + a_4 Q + \varepsilon \qquad (3.16)$$

where

g = a weighted average of growth in dividend due to retention $\alpha\rho^*$ and stock financing sv[3]

se = percentage of electricity sales to total sales

Q = an index of the quality of earnings

Next the average leverage-free rate of return on common stock was calculated as

$$a_o^* = \hat{a}_o + \hat{a}_2 \bar{g} + \hat{a}_3 \overline{se} + \hat{a}_4 \bar{Q} \qquad (3.17)$$

where

$\bar{g}$, $\overline{se}$, and $\bar{Q}$ are sample average values

Table 3.4 reports the results. In all years the difference between $(a_o^* = \hat{a}_1)$ and i is significant at the 5 percent level of significance. Gordon also reformulated his statistical equation to allow for varying risk; that is, different ρ, among the sample firms. The results were not improved. The quantity $\hat{a}_o - \hat{a}_1$ remained significantly larger than i.

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[3] The weights used were estimated by a regression of dividend yield on a number of variables including an infinite horizon growth rate $g_1 = \alpha\rho^* + sv$ and a smoothed earnings per share growth rate g_2.

Table 3.4

Test of the PCCM Leverage Theorem

Year	a_o^*	$a_o^* - \hat{a}_1$	i	t value $(a_o^* - \hat{a}_1) - i$
1958	.0866	.0815	.0422	10.03
1959	.0918	.0861	.0471	9.84
1960	.0877	.0836	.0449	9.03
1961	.0801	.0749	.0450	6.10
1962	.0842	.0779	.0431	8.80
1963	.0842	.0776	.0439	9.16
1964	.0865	.0824	.0446	9.05
1965	.0880	.0828	.0475	8.40
1966	.0965	.0923	.0546	8.88
1967	.0999	.0960	.0640	5.28
1968	.0939	.0892	.0675	4.16

Source: Gordon [26].

Test of the Traditional Leverage Thereom

To test the traditional leverage theorem for a public utility, Gordon changed the leverage variable from D/S to B/E. Although ρ is still estimated by $\hat{a}_o$, there is no theoretical basis requiring $\hat{a}_1$ be equal to $\rho - i$. Rather, according to the traditional view, it should lie between 0 and $\rho - i$, depending upon the average coupon rate relative to the existing yield on debt. When the historical debt-to-equity ratio was used as the leverage variable rather than the market value debt-to-equity ratio, $\hat{a}_1$ was positive, but significant in only five of the eleven years. Once again Gordon calculated an estimate of the leverage-free rate of return on common stock by equation (3.17) and compared the quantity $(a_o^* - i)$ to $\hat{a}_1$, the estimate of γ in equation (3.13). Gordon concluded that since $\hat{a}_1$ was smaller by amounts significant at the 1 percent level or better, his results were in agreement with the traditional theory (as he defined it for electric utilities).

The Imperfect Market Model

Gordon based his imperfect market model on the same hypothesis of Brigham and Gordon [13]. That is, k increases with $(g = \alpha\rho^*)$ by increasing amounts such that

$$k = \alpha\rho^* + \frac{1}{a_o e^{a_1 \alpha\rho^*}} \qquad (3.18)$$

where $1/a_o$ is the yield at which a share would sell with zero growth and $a_1 > 1$. Substitution of (3.18) into equation (3.5) gives the present value of a share of common stock as

$$P = a_o(1 - \alpha)(1 - \tau)xe^{a_1\alpha\rho^*} \qquad (3.19)$$

To allow for financial leverage and common stock financing additional exponential variables were included resulting in

$$P = a_o(1 - \alpha)(1 - \tau)xe^{a_1\alpha\rho^*} e^{a_2B/E} e^{a_3vs} e^{a_4s^2} \qquad (3.20)$$

where

v = the fraction of funds provided during period t that accrues to shareholders at the beginning of period t

s = funds raised from the sale of common stock as a fraction of existing common equity.

Gordon tested seven different statistical formulations of (3.20) by pooling the eleven years of data and including $\ln(\frac{1}{I})$ as an additional variable to account for changing interest rates. The selection of

$$\ln(\frac{P}{E}) = \ln[a_o] + a_1\ln(\frac{d_1}{E}) + a_2\alpha\rho^* + a_3vs$$

$$+ a_4ge + a_5\frac{B}{E} + a_6s^2 + a_7se + u \qquad (3.21)$$

as the best statistical equation was based on the reasonableness of the coefficients, and Gordon's belief in the virtue (for a utility) of using book value of equity as a deflator. Additional independent variables in (3.21) are ge defined as the excess of the smoothed rate of growth in earnings, and se defined above. Table 3.5 reports the results of cross-section regressions for eleven years. Coefficients $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ are all significant with the correct sign in every year. The others are generally significant with the correct sign.

Table 3.5

Cross-Section Estimates of the Parameters of an Imperfect Capital Markets Model

$\ln(P/E) = \ln(a_0) + a_1 \ln(d_1/E) + a_2 bc^* + a_3 vs + a_4 ge + a_5 B/E + a_6 s^2 + a_7 se + u$

Year	$\ln(a_0)$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{a}_4$	$\hat{a}_5$	$\hat{a}_6$	$\hat{a}_7$	$\bar{R}^2$ S.E.E.
1958	1.508	.463 (.090)	7.47 (1.09)	16.31 (2.10)	1.14 (.64)	-.064 (.030)	-28.36 (8.60)	.042 (.066)	.918 (.0734)
1959	1.456	.556 (.098)	8.97 (1.05)	17.69 (2.07)	1.86 (.87)	-.048 (.041)	-42.12 (12.06)	.191 (.081)	.893 (.0857)
1960	1.521	.589 (.096)	10.02 (1.29)	20.60 (2.23)	3.02 (.86)	-.006 (.048)	-53.80 (15.07)	.190 (.077)	.905 (.0889)
1961	1.804	.670 (.103)	14.94 (1.76)	13.47 (2.89)	.78 (1.01)	-.032 (.063)	-22.43 (25.21)	.218 (.088)	.846 (.1013)
1962	2.301	.859 (.084)	11.98 (1.21)	8.54 (2.18)	2.45 (1.25)	-.134 (.055)	3.50 (25.16)	.203 (.082)	.850 (.0948)
1963	2.302	.800 (.075)	12.65 (1.20)	12.04 (2.14)	4.77 (1.37)	-.085 (.047)	-35.73 (26.66)	.136 (.080)	.882 (.0862)
1964	2.431	.849 (.075)	11.28 (1.22)	9.16 (1.72)	5.19 (1.51)	-.090 (.047)	4.23 (27.30)	.210 (.079)	.878 (.0884)
1965	2.033	.799 (.077)	13.35 (1.33)	12.62 (4.06)	4.71 (1.63)	-.039 (.051)	-2.10 (3.81)	.237 (.077)	.877 (.0905)
1966	1.689	.835 (.074)	15.52 (1.58)	12.65 (3.83)	6.72 (1.51)	-.027 (.050)	.590 (3.57)	.344 (.075)	.915 (.0869)
1967	1.760	.658 (.079)	7.67 (2.13)	33.65 (4.73)	4.77 (1.61)	-.046 (.047)	-14.53 (4.04)	.106 (.089)	.885 (.0976)
1968	2.010	.652 (.068)	6.23 (1.63)	14.71 (2.78)	.84 (1.31)	-.096 (.034)	-5.10 (2.07)	.121 (.070)	.886 (.0790)

Source: Gordon [26].

Note: The numbers in parentheses are the standard errors of the parameter estimates.

Critique of Gordon's Study

In spite of Gordon's assertion that equation (3.21) did an excellent job of explaining market-to-book ratios among utility stocks, the high R^2 values are perhaps misleading. Correlation among the independent variables was not examined. However, it is likely that high positive correlation exists between the growth variables $\alpha\rho^*$, vs, s^2, and ge. That additional explanatory variables appear in the statistical equation, but not the theoretical model (see equation (3.20)) is also disturbing. Finally, it must be noted that component costs are not directly estimated by the statistical equation. Rather, from (3.20) Gordon derives equations for the cost of debt, cost of retention capital, cost of stock financed capital, and finally an average cost of capital. Nevertheless, Gordon's monograph remains a most detailed exposition on the determinants of value for electric utilities.

Litzenberger and Rao: Estimates of the Marginal Rate of Time Preference and Average Risk Aversion of Investors in Electric Utility Shares: 1960-66

Litzenberger and Rao [32] developed an econometric model of the valuation of electric utility shares under uncertainty. Their model is similar to the M & M all-equity growth model developed in [38], but it differs in that individual firm business risk enters the model. Assumptions underlying the Litzenberger-Rao model are: (1) there exists a perfectly competitive capital market dominated by a large number of risk-averse investors each with sufficient funds for complete diversification; (2) each investor has a utility function definable over the first two moments of the distribution of portfolio rates of return; (3) investors are characterized as possessing identical probabilistic expectations of future earnings for each firm; and (4) investors may borrow or lend freely at the pure rate of interest.

Development of the Model

Given these assumptions, it can be shown that the required rate of return on a share of common stock is[4]

$$k_i = r_o + [k_m - r_o] \frac{\sigma_{im}}{\sigma_m^2} \tag{3.22}$$

where

r_o = the riskless rate of interest

k_m = the required rate of return on the market portfolio of risky assets

σ_{im} = the covariance between the rates of return on the ith security and the market portfolio

$\sigma_m 2$ = the variance of the rate of return on the market portfolio

It is possible, of course, to also write (3.22) as

$$k_i = R_o + \frac{R_m - R_o}{\sigma_m} \cdot \rho_{im}\sigma_i \tag{3.23}$$

In the absence of growth it can be shown that the market value of a share of common stock is

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[4] The seminal work in portfolio and capital market theory is attributed to Markowitz [34], Tobin [52], Sharpe [51], Lintner [31], and Mossin [44].

$$P_i = \frac{(1 - \tau)\overline{x}_i}{k_i} \tag{3.24}$$

Expression (3.24) combined with the homogeneous expectations of investors permits equating the nondiversifiable standard deviation, $\rho_{im}\sigma_i$, of (3.23) with

$$\frac{\rho_{im}\sigma_i(Y)}{P_i} \tag{3.25}$$

Substituting (3.25) into (3.23), then replacing k_i of (3.24) with expression (3.23) and solving for P_i gives

$$P_i = \frac{(1 - \tau)\overline{x}_i - \lambda\rho_{im}\sigma(1 - \tau)x_i}{r_o} \tag{3.26}$$

The term $\lambda = [k_m - r_o]/\sigma_m$ is the marginal required return per unit of nondiversifiable standard deviation; thus, $\lambda\rho_{im}\sigma(1 - \tau)x_i$ is the additional earnings required as a premium for undiversifiable risk. The right-hand side of (3.26) then is a certainty equivalent value for security i.

To include growth in the valuation model, Litzenberger and Rao used the M & M growth component developed in [39], i.e.,

$$G_i = \frac{\Delta BV_i(\rho^* - k_i)}{k_i(1 + k_i)} T \tag{3.27}$$

where

ΔBV_i is the expected dollar amount per share of equity investment for firm i during the current year

ρ_i^* is the expected after-tax rate of return on equity investment assumed equal to the existing rate of return, $(1 - \tau)x_i/E_i$

Adding G_i to (3.26) gives

$$P_i = \frac{(1 - \tau)\bar{x}_i - \lambda\rho_{im}\sigma(1 - \tau)x_i}{r_o} + \frac{\Delta BV\ (\rho_i^* - k_i)}{k_i(1 + k_i)}\ T \quad (3.28)$$

Statistical Equations

From equation (3.28) a general statistical equation was formed as

$$P_i = \gamma_1(1 - \tau)\bar{x}_i + \gamma_2 s[(1 - \tau)x_i] + \gamma_3\left[\frac{\Delta BV_i(\rho_i^* - k_i)}{k_i(1 + k_i)}\right] + \varepsilon_i \quad (3.29)$$

where

$\gamma_1 = 1/r_o$

$\gamma_2 = \lambda\rho_{im}/r_o$

$\gamma_3 = T$

$s[1 - \tau)x_i]$ = the sample standard deviation of earnings per share for firm i

ε_i = a random disturbance term

To be assured that Var (u_i) was constant for the sample, Litzenberger and Rao found it necessary to deflate equation (3.29). One alternative was to divide (3.29) by BV, the recorded value of common stock, thus forming

$$\frac{P_i}{BV_i} = a_1\rho_i^* + a_2 s(\rho_i^*) + a_3\frac{G(BV_i)(\rho_i^* - k_i)}{k_i(1 + k_i)} + u_i \quad (3.30)$$

where

BV_i = the recorded per share value of equity for firm i

$s(\rho_i^*)$ = the sample standard deviation of the ith firm's rate of return on investment

$G(BV_i)$ = the average change in the ith firm's book equity per share as a percentage of book equity

$u_i = \varepsilon_i / BV_i$

Equation (3.29) was also deflated by the market price per share and solved for the earnings yield $\overline{\pi}^{\tau}(1 - \tau)x_i/P_i$. This transformation resulted in

$$\overline{\pi}^{\tau} = b_o + b_1 s(\pi_i) + b_2 \frac{G'[BV_i](\rho_i^* - k_i)}{k_i(1 + k_i)} + u_i' \qquad (3.31)$$

where

$b_o = 1/\gamma_1 = r_o$

$b_1 = -\gamma_2 r_o = \quad _{im}$

$b_2 = -\gamma_3 r_o = Tr_o$

$s(\pi_i)$ = the sample standard deviation of the ith firm's earning's yield

$G'(V_i)$ = the average change in the ith firm's book equity per share as a percentage of market value

$u_i' = \varepsilon_i / P_i$

Results for equation (3.30). Table 3.6 reports the regression results for equation (3.30). Of particular importance is $1/\gamma_1$, the marginal rate of time preference, ranging between .05 and .06 over the sample period. The required return per unit of nondiversifiable risk λ ranges between .24 and 1.33, and the value of T is consistent with earlier findings by Malkiel [33]. The average required rate of return for a share of stock is calculated as[5]

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[5] Equation (3.32) follows M & M [41] where the required rate of return for a levered share is expressed as $k^L = (V/S)k^U$. It follows that $\sigma(r_i^U) = (1 - L_i)\sigma(r_i^L)$ where L_i is the market debt-to-total value ratio for firm i.

Table 3.6

Regression Results for Equation (3.30)

Year	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	R^2	d^+	I	$\hat{\rho}_{im}$	T	$\hat{\lambda}$	$\hat{k}$
1960	16.7 (0.74)	-22.1 (7.5)	1.7 (0.61)	0.55	2.10*	0.060	1.33	1.7	1.67	0.069
1961	18.7 (1.13)	-12.0 (11.2)	2.3 (0.80)	0.52	1.91*	0.053	0.64	2.3	0.80	0.058
1962	17.8 (0.73)	-11.4 (9.0)	1.5 (0.79)	0.57	2.12*	0.056	0.65	1.5	0.81	0.061
1963	19.8 (0.79)	-16.2 (10.6)	2.4 (1.2)	0.56	2.54*	0.051	0.82	2.4	1.03	0.056
1964	20.2 (0.82)	-17.8 (11.9)	3.7 (1.5)	0.57	2.01*	0.050	0.70	3.7	0.88	0.054
1965	19.9 (0.83)	-4.8 (10.9)	5.7 (2.6)	0.55	2.10*	0.050	0.24	5.7	0.30	0.051
1966	17.3 (0.36)	-10.4 (4.9)	3.6 (2.7)	0.50	1.94*	0.057	0.59	3.6	0.93	0.061
Overall	18.9 (0.25)	-9.8 (2.3)	1.0 (0.32)	0.44	2.00*	0.053	0.51	1.0	0.84	0.056

Source: Litzenberger and Rao [32].

+ Durbin-Watson Statistic.

* The null hypothesis of no serial correlation is accepted at the 0.05 level.

$$k_i = r_o + \lambda\rho_{im}(1 - L_i)s(r_i) \qquad (3.32)$$

Litzenberger and Rao also suggested that the marginal cost of capital could be calculated by the M & M cost of capital specification.

Results for equation (3.31). Table 3.7 contains the results of regression equation (3.31). It should be noted that equation (3.31) was primarily used to check for biases in the regression of (3.30). As borne out by the numbers in Table 3.7, the estimated coefficients for the earnings yield model are less stable over time than the estimates for equation (3.30). To stabilize the coefficients Litzenberger and Rao pooled the seven years of data, constraining $\lambda\rho_{im}$ to a constant and used dummy variables to allow I and T to vary each year.

Results of the pooled regression appear in Table 3.8. The marginal required return per unit of standard deviation was .54 and significant over the seven years. The marginal rates of time preference were stable ranging from .053 and .061. The coefficients for the growth variable were significant and negative for each year varying between 1.9 and 13.2 years.

Summary of the Litzenberger and Rao Study

The significance of the Litzenberger-Rao study is the provided link between the assets of a firm and the valuation of that firm in the capital market. The next important contribution of Litzenberger and Rao though, is the explicit recognition of interfirm differences in operating (business) risk.

Table 3.7

Regression Results for Equation (3.31)

Year	$\hat{b}_o$ (I)	$\hat{b}_1$ ($\lambda\rho_{im}$)	$\hat{b}_2$	R^2	d†	T
1960	0.059	0.06 (0.34)	-0.14 (0.027)	0.247	2.09	2.3
1961	0.053	0.63 (0.38)	-0.17 (0.027)	0.36	1.93	3.2
1962	0.057	0.35 (0.40)	-0.15 (0.035)	0.25	2.12	2.6
1963	0.054	0.49 (0.39)	-0.22 (0.041)	0.33	2.38	4.1
1964	0.054	0.36 (0.47)	-0.27 (0.054)	0.30	2.18	5.0
1965	0.058	0.08 (1.23)	-0.68 (0.266)	0.09	2.02	11.7
1966	0.061	0.45 (0.27)	-0.34 (0.143)	0.08	1.78	5.6
Overall	0.055	0.34 (0.11)	-0.13 (0.015)	0.11	2.04	2.4

Source: Litzenberger and Rao [32].

† The null hypothesis of no positive serial correlation is accepted at the 0.05 level.

Table 3.8

Regression Results for Equation (3.31)(Pooled Data)

Year	$\hat{a}_o$ (I)	$\hat{a}_1$* ($\lambda\rho_{im}$)	$\hat{a}_2$	R^2	T	b*
1960	0.061	0.54 (0.17)	-0.11 (0.024)	0.30	1.9	0.68
1961	0.054		-0.17 (0.029)		3.2	
1962	0.056		-0.16 (0.035)		2.9	
1963	0.053		-0.23 (0.051)		4.3	
1964	0.054		-0.29 (0.062)		5.4	
1965	0.056		-0.74 (0.130)		13.2	
1966	0.061		-0.38 (0.103)		6.2	

Source: Litzenberger and Rao [32].

* $\hat{a}_1$ is constrained to be an intertemporal constant.

McDonald: Required Return on Public Utility Equities: A National and Regional Analysis, 1958-69

The final empirical cost of capital study to be reviewed is McDonald [36]. The theoretical model used is M & M's formula for the pure equity firm; debt is ignored. Thus, McDonald wrote for the market value of a firm's equity

$$S = \frac{(1 - \tau)\overline{X}}{k} + \frac{\overline{I}(\rho^* - k)}{k(1 + k)} T \qquad (3.33)$$

or, expanding the second term on the right-hand side of (3.33)

$$S = \frac{(1 - \tau)\overline{X}}{k} - \frac{\overline{I}T}{1+k} + \frac{\rho^*\overline{I}T}{k(1 + k)} \qquad (3.34)$$

Statistical Analysis and Results

McDonald wrote a statistical equation for (3.34) as

$$S = a_1(1 - \tau)\overline{X} + a_2\overline{I} + a_3\rho^*\overline{I} + u \qquad (3.35)$$

where

$a_1 = 1/k$

$a_2 = -T/(1 + k)$

$a_3 = -a_1a_2 = T/k(1 + k)$

Because (3.35) is intrinsically nonlinear in its parameters, McDonald used nonlinear least squares regression to calculate maximum likelihood estimates of a_1 and a_2. A detailed synopsis of nonlinear regression is in Chapter IV.

The sample. To verify that (3.34) correctly models total equity value for a utility, McDonald drew a sample of 102 electric utilities from the Compustat Utility file. A test proposed by Gonedes [23] was used to satisfy McDonald that the sample of firms was homogeneous in business risk.[6] Expected values of $(1 - \tau)\overline{X}$ and $\overline{I}$ were estimated for 1958-1969 inclusive by a 3 year distributed lag model. The product $\rho^*\overline{I}$ was taken as

$$\rho^*\overline{I} = [\Lambda(1 - \tau)\overline{X}/\overline{I}\}\cdot\overline{I} = \Lambda(1 - \tau)\overline{X} \qquad (3.36)$$

Problems of heteroscedasticity were avoided by deflating all variables by A, the book value of total assets. Thus, the statistical equation in its final form was

$$\frac{S}{A} = a_1\frac{(1 - \tau)\overline{X}}{A} + a_2\frac{\overline{I}}{A} + a_1a_2\frac{\Delta(1 - \tau)\overline{X}}{A} + \varepsilon \qquad (3.37)$$

Table 3.9 contains estimates of a_1 and its inverse, the required rate of return on equity for each of the 12 years. Also reported for each year is: the average earnings yield on utility stocks, yields on utility preferred stocks and bonds, and the percentage change in the GNP price deflator. All values of $\hat{a}_1$ are highly significant, and each inverse of $\hat{a}_1$, when compared to the other yields reported in the table, appears to be a reasonable estimate for k. Also, the estimates of k are close to those of Litzenberger and Rao (see Tables 3.6-3.8).

Summary of the McDonald Study

The major importance of McDonald's study of utility equity value is the explicit recognition of the nonlinear growth term in the M & M growth model. Because of the restricted coefficients in (3.35) the model is underidentified. Recall that M & M impacted

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[6] A review of Gonedes is below.

Table 3.9

Estimates of Required Return on Utility Equities and Comparison with Some Published Market Yields

Year	$\dot{a}_i$	R^2	$\frac{k}{100/\dot{a}_i}$	Earnings Price Ratio *	PFD. Stock Yield Med. Grade*	Bond Yield (Rating Aa)*	% Change in GNP Price Deflator**
1958	15.53 (0.32)	0.60	6.4%	6.3%	4.8%	3.9%	1.65%
1959	14.90 (0.33)	0.54	6.7	5.8	5.0	4.6	1.30
1960	17.23 (0.41)	0.33	5.8	5.9	5.1	4.5	1.50
1961	21.03 (0.50)	0.51	4.8	4.8	4.9	4.5	1.04
1962	19.23 (0.26)	0.55	5.2	5.2	4.7	4.4	1.12
1963	19.71 (0.35)	0.57	5.1	4.9	4.6	4.3	1.70
1964	20.74 (0.48)	0.56	4.8	5.0	4.7	4.4	1.57
1965	18.81 (0.38)	0.55	5.3	5.1	4.7	4.5	1.73
1966	15.76 (0.33)	0.51	6.3	6.1	5.4	5.3	3.22
1967	13.89 (0.45)	0.44	7.2	6.5	5.8	5.7	3.21
1968	13.97 (0.38)	0.55	7.2	6.8	6.3	6.4	3.38
1969	11.06 (0.38)	0.51	9.0	7.3	6.6	7.0	4.74
Average (1958-1969)			6.2%	5.8%	5.2%	5.0%	2.3%

Source: McDonald [36].

Note: Standard errors are in parentheses.

* Moody's Public Utility Manual.

** Economic Report of the President.

everything in the growth term into the coefficient except $\bar{I}$ which they measured as $\overline{\Delta A}$. For M & M to estimate the value of T, they first had to propose a value for ρ^*. By recognizing explicitly a_1, a_2 in the regression equation an estimate of T could have been calculated directly from the coefficients. Unfortunately, estimates for a_2 were not reported leaving the importance of growth in equation (3.33) unknown.

Equivalent Risk Class Studies

Testing the effect that financial leverage has on firm value requires a sample homogeneous in business risk. Except for the studies of Litzenberger and Rao [32] and McDonald [36], industrial classification of firms was assumed to satisfy this requirement. Litzenberger and Rao accounted for different degrees of business risk within the context of their model. McDonald, using a test proposed by Gonedes [23], concluded that his sample of utilities was homogeneous.

That industrial classification of firms provides homogeneous risk classes has been tested by Wippern [56] and Gonedes [23]. Wippern's study used the parametric analysis of variance model to (1) reject the null hypothesis that business risk is equal among eight industry groups, (2) conclude that there is, in general, as much variation within groups as among those groups, and (3) conclude that the rejection of the equal-means hypothesis was entirely due to one industry, electric utilities. The findings of Gonedes corroborated Wippern's conclusion that groups of firms homogeneous in business risk cannot be defined by industrial classification.

However, his tests of the equivalent risk class hypothesis were conducted using a nonparmetric model, the Kruskal-Wallis One-Way Analysis of Variance by Ranks.

Wippern: A Note on the Equivalent Risk Class Assumption

The business risk surrogate selected by Wippern was

$$s_i = \text{Antilog } \sqrt{\text{MSE}_i} \tag{3.38}$$

in which MSE_i is the mean square error for firm i of the 10 year trend

$$x_i(t) = x_i(o)e^{g \cdot t}u_i(t) \tag{3.39}$$

where

$x_i(t)$ = earnings before interest per share for firm i in period t

g = continuous rate of growth

$u(t)$ = random disturbance term in period t for firm i

Evaluation of s_i as a risk surrogate. Because earnings before interest and taxes has been divided by the number of shares outstanding, s_i fails to isolate that part of the earnings stream which varies only due to business risk. Indeed, scaling $x_i(t)$ by $N_i(t)$ allows the financing decision to intrude as an additional source of variation.

The sufficiency of using the standard deviation to adequately describe the distribution of x_i is more complicated. First define x' as $\ln(x)$. Now if $\Phi(x')$ is the unknown distribution of $\ln(x')$, then s_i may or may not be sufficient to describe $\Phi(x')$. For example, if $\Phi(x')$ is normal, then $\Phi(x')$ is lognormal, and s_i reflects the presence of skewness in $\Phi(x)$. However, the possibility of negative $x(t)$ makes the existence of $\Phi(x')$ in general unlikely. But if higher moments of $\Phi(x)$ exist, then testing for equality among standard deviations of each industry is necessary but not sufficient to conclude equal populations. It is well documented (cf. [2], [3], [29, pp. 246-247], and [37]) that investors consider the higher moments of a distribution when selecting risky assets.

Statistical analysis. Defining $s_{\cdot j}$ as an unbiased estimator of the industry average standard deviation σ_j (mean percentage variability) of the n_j firms in industry group j, the null hypothesis

$$H_o: \quad \sigma_1 = \sigma_2 = \ldots \sigma_8 \tag{3.40}$$

was rejected at the one percent level of significance. Next, Wippern used Scheffe's method of multiple comparisons to test the null hypothesis that simultaneously[7]

$$H_o: \quad L_k = \sum_{j=1}^{2} C_j \sigma_j = 0, \text{ for all } k \tag{3.41}$$

where

L_k = the kth pairwise contrast or difference in a family of possible contrasts (eight industry groups indicate 28 possible contrasts)

C_j = a coefficient such that $C_1 = 1$ and $C_2 = -1$.

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[7] The Scheffe method of multiple comparisons is explained in Chapter V. Also, see Scheffe [50, pp. 66-72].

Table 3.10 lists the number of firms, the average s_i, and the range of s_i for each industry. Only 4 of the 28 pairwise differences were rejected as significantly different than zero, all occurring when the pairwise comparison included the electric utility.

A glance at the table reveals that the average percentage variability of electric utilities was much less ($s_{.3} = .046$) than for any other industry group and according to the multiple comparisons test, significantly different from: machinery ($\alpha = .99$), oil ($\alpha = .95$), drugs ($\alpha = .95$), and cement ($\alpha = .75$). Thus, Wippern concluded that grouping firms by industrial classification did not in general result in homogeneous and unequal risky groups.

Evaluation of the statistical analysis. An unfortunate exclusion in Wippern's analysis is the lack of evidence justifying the use of the parametric model. Instead, the general robustness of the F test and its related analyses is relied upon for departures from normal distributions and unequal variances. The robustness of the F test, however, has several characteristics not considered by Wippern. One, it can be shown that the Scheffe multiple comparisons method is not seriously affected by lack of normality provided the sample sizes are not extremely small. Two, if the variances of each factor level are unequal, the F test and its related analyses are only slightly affected provided all sample sizes are equal.[8] Table 3.10 reveals that several of the sample sizes were quite small;

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[8] The Scheffe method of multiple comparisons is explained in Chapter V. Also, see Scheffe [50, pp. 66-72].

Table 3.10

Percentage Variability of EBIT Per Share

j	Industry	n_j	$s_{\cdot j}$	Range of s_{ij}
1	Baking	4	0.133	0.063-0.296
2	Cement	5	0.216	0.123-0.276
3	Electric Utilities	17	0.046	0.024-0.066
4	Ethical Drugs	6	0.249	0.212-0.455
5	Machinery	6	0.275	0.170-0.431
6	Oils	8	0.227	0.114-0.351
7	Paper	10	0.159	0.039-0.449
8	Rubber	5	0.141	0.092-0.259

Source: Wippern [56].

furthermore, they vary considerably. Electric utilities have more than twice the observations of 6 of the other 7 industries. The effect of such heterogeneity in sample size and variance is to increase the level of significance.

A related problem with Wippern's analysis is the power of his tests. Power is the probability that the null hypothesis is correctly rejected. It depends upon the level of significance, the number of observations, and a noncentrality parameter, inversely related to known population variance. For a level of significance, power increases as the number of observations increase and as the population variance decreases. With so few observations and so wide a range of s_i for each industry, the power of Wippern's test must be considered low.

Gonedes: A Test of the Equivalent Risk Class Hypothesis

The surrogate for business risk devised by Gonedes [23] was the relative deviation of a firm's annual rate of growth in net operating income from the firm's 10 year compound rate of growth in annual net operating income. Thus business risk was indicated by

$$BR_i(t) = \frac{\dot{X}_i(t) - G_i(X)}{G_i(X)} \tag{3.42}$$

where

$\dot{X}_i(t)$ = the annual growth rate of EBIT during period t (t = 1958-67) for firm i

$G_i(X)$ = the compound growth rate of EBIT over the 10 year period

Evaluation of $BR_i(t)$. Expression (3.42) implies that realized EBIT during the 10 years equaled investors' expectations. To see this, note that $G_i(X)$ is the geometric average of $\dot{X}_i(1)$, $\dot{X}_i(2)$, . . . , $\dot{X}_i(T)$; the calculation of $G_i(X)$ thus requires knowledge of annual growth rates unknown at time t. Since $BR_i(t)$ is expressed as an annual deviation, a better point from which to measure the deviation would have been what the annual level of EBIT growth was on average, not the compound level of growth for 10 years.

Statistical analysis. Gonedes used the Kruskal-Wallis One-Way Analysis of Variance by Ranks to test the following hypotheses:

1. There are no significant differences among firms within a given industry with respect to business risk.
2. Industry groups do not evidence significant differences with respect to business risk.

Testing of the hypotheses proceeded along the following lines. First, random samples of eight different industries were drawn and $BR_i(t)$ were calculated. To test the first hypothesis, 100 observations for each industry (ten firms over ten years) were ranked with the smallest value of $BR_i(t)$ being assigned a rank of one, the next to the smallest a two, and so on to 100. Next, the sum of the ranks of $BR_i(t)$ for each firm was computed over the 10 years and compared with the sum of the ranks over the same 10 years for the other 9 firms in an industry. In general, the Kruskal-Wallis test determines whether these sums of ranks are so disparate that they are not likely to belong to samples drawn from the same population. Specifically, if J independent samples are from the same population, then it can be shown that the Kruskal-Wallis statistic is distributed chi-square with J - 1 degrees of freedom. The Kruskal-Wallis statistic is defined generally as

$$H = \frac{12}{n(n + 1)} \sum_{j=1}^{J} \frac{SR_j}{n_j} - 3(n + 1) \qquad (3.43)$$

where

J = number of samples

n_j = number of cases in the jth sample

$n = \sum n_j$, the number of cases in all samples combined

SR_j = sum of ranks in jth sample

For six of the eight industry samples the null hypothesis of intraindustry homogeneity was rejected.

To test the hypothesis of interindustry homogeneity, composite data for each industry was calculated. The resulting eighty business risk surrogates were then ranked as before and the H statistic calculated. The null hypothesis of no significant differences between industries was rejected.

Evaluation of the statistical analysis. Gonedes used the Kruskal-Wallis model because he was unsure of the appropriateness of the F test. Specifically, he questioned whether the assumptions of the parametric model were satisfied by Wippern's sample data. If the assumptions of the parametric model are satisfied, nonparametric tests are always less powerful than parametric tests on the same data. Nonparametric tests require only that selection of any one case from a population does not bias the chances of any other case being included in a sample from that population. By avoiding the additional assumptions of the parametric model--normality and homoscedasticity--more generality is achieved, albeit with a loss of power. Clearly, if all assumptions of the parametric model are satisfied with the sample data, then that model should be used. However, even when the assumptions are fully met Andrews [1] has shown the power-efficiency of the Kruskal-Wallis test to be 95.5 percent that of the F test.[9]

Appropriateness of the Kruskal-Wallis model. A more general statement of the hypothesis tested by Gonedes is

$$H_o: \quad x_1(BR) = x_2(BR) = . . . = x_n(BR) \qquad (3.44)$$

where

$x_i(BR)$ = the cumulative distribution function for population i

It is important to understand that the Kruskal-Wallis model tests only for equal medians of x_i. To conclude also that the populations are homogeneous, all other parameters of $x_i(BR)$ (variance, skewness, etc.) must be identical. Thus, while Gonedes correctly rejected the null hypothesis for 6 of 8 industries, it cannot be concluded the other 2 industries (textiles and retail department stores) were homogeneous in business risk. The point is important; McDonald [36] (reviewed above) replicated Gonedes' test for a sample of 87 electric utilities and concluded that the firms came from the same population with respect to business risk. McDonald's conclusion was correct only if the distributions of $BR_i(t)$ for all 87 were identical.

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[9] Power-efficiency indicates the percentage increase in sample size necessary to make one test just as powerful as another. For the results of some tests on the power-efficiency of the Kruskal-Wallis test, see Andrews [1, pp. 724-30].

Summary

This chapter has been devoted principally to the review of cost of capital studies. Each of the studies reported numbers identified as the cost of equity capital. Marginal cost of capital estimates were reported only by Miller and Modigliani [39]; Litzenberger and Rao [32] suggested that their estimates could be used in conjunction with the M & M formula (see equation (2.36)) to produce marginal cost estimates.

The studies are distinctive in their diverse methodologies for calculating estimates of ex ante variables in the several models. The methods included arithmetic averages, distributed lag models, exponential trend, multiple regression models, and the instrumental variable approach of M & M.

Recognition of firm or share value growth was important to each of the studies. The same growth model proposed and used by M & M was also adopted in the models of both Litzenberger and Rao and McDonald. Growth in the Gordon study was based on the growth in dividends due to internal and external equity financing. Only the Litzenberger and Rao study provided unrestricted estimates of the duration of growth. The growth duration estimates reported by M & M were based on an assumed rate of return in the growth variable. The nonlinearity of the growth term was explictly recognized by

McDonald who adopted an iterative least-squares routine for estimating coefficients of his model. Unfortunately, McDonald chose not to report the growth coefficient of his model.

Litzenberger and Rao's major contribution was to treat each firm as a separate risk class. Both M & M and Gordon assumed that electric utilities formed a homogeneous population with respect to business risk. McDonald used Gonedes' methodology [23] to support his view that his sample of electric utilities formed a homogeneous business risk class. That industrial classification cannot be relied upon to group firms into business risk classes was pointed out in the review of Wippern [56] and Gonedes [23].

Each of the cost of capital studies reported what can be considered reasonable estimates of the cost of equity capital. However, even accepting the obtained numbers as evidence of each model's worth leaves the task of calculating an average cost of capital.

CHAPTER IV

DIRECT COST OF CAPITAL MODEL: STRUCTURAL AND STATISTICAL EQUATIONS

Chapter II pointed out the more robust characteristics of the M & M cost of capital model vis-a-vis the traditional model. Also noted in Chapter II was the misspecification of the growth term in the M & M valuation equation for the levered firm with growth potential. Three of the four empirical studies reviewed in Chapter III used the M & M formulation of growth in their estimation of the cost of equity capital; ignored was the fact that the original growth formula was derived for a pure equity firm. However, even acceptance of the cost of equity estimates reported in Chapter III requires a combination with other component costs be made to obtain an overall or average cost of capital. For regulators this combining process remains a topical issue. For example, should the book or the market values of capital components be used to calculate a weighted average? For electric utilities, is embedded (historical) interest or marginal interest the more appropriate cost of debt? Should the cost be tax adjusted as hypothesized by the traditional model, or is the tax benefit resulting from the deductibility of interest payments already reflected in the cost of equity capital?

Direct Cost of Capital Model

In an efficient capital market investor perceived rewards and risks of a firm's securities--bonds, preferred stock and common stock--will be reflected in the total values of these securities. Thus, it seems a waste of energy to ferret out the specific values, benefits, and risks of each component of capital, calculate the component costs and average them together (somehow) when the purpose of the exercise is to calculate the overall or average cost. Why not estimate it directly and not by its component parts? There are several approaches one can take. For this study a partial equilibrium growth model is examined. To implement this approach, requires only that the constant growth valuation model be applied to the firm as an entity rather than restricting the model's use exclusively to the valuation of the firm's equity.

Assumption of the Model

The capital markets are assumed to be in equilibrium with rates of return on various securities given to investors and firms. The question of how such rates are determined is ignored.[1] To properly formulate the model, two additional assumptions are needed.

Perfect capital markets. Several characteristics describe a perfect capital market. One, no individual or firm is large enough to affect prices; two, there is complete and costless information available to everyone; three, all individuals and firms borrow and lend on exactly the same terms; four, there are no transaction costs; and five, there is no differential among tax rates for investors or types of income.

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[1] See Sharpe [51], Lintner [31], and Mossin [44]. For a clear exposition of risk, rates of return, and market equilibrium see Fama and Miller [22, Chapter 7].

Rational investors. Investors are presumed to always act so as to maximize their end of period wealth. Furthermore, investors are indifferent as to whether increments to their wealth take the form of cash payments or increased market value of shares.

Construction of the Model[2]

Let $\overline{Div}(t)$, $\overline{Int}(t)$ and $\overline{Pfd}(t)$ respectively represent investors' consensus expectation of the firm's total payment of common stock dividends, interest, and preferred dividends at the end of period t. In addition, symbolize the firm's expected net investment in additional assets at the end of period t by $\overline{I}(t)$, total expected external funds raised at the end of period t by $\overline{NF}(t)$, and the firm's total market value (debt, preferred stock, and common stock) at the beginning of t by V(t). Then Z the expected marginal return from investing in the firm is

$$Z = \frac{\overline{Int}(t) + \overline{Pfd}(t) + \overline{Div}(t) + [\overline{V}(t+1) - \overline{NF}(t) - V(t)]}{V(t)} \tag{4.1}$$

in which it should be noted that part of the firm's value at the end of period t or the beginning of t + 1 belongs to the present security holders of the firm, i.e., the firm's total expected value $\overline{V}(t+1)$ less the market value of the new securities issued, $\overline{NF}(t)$, during t. Equation (4.1) can be rearranged as

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[2] The model was suggested to me by Fred Arditti. Part of the development of the model in this section is contained in [6].

$$V(t) = \frac{1}{1+Z}[\overline{Int}(t) + \overline{Pfd}(t) + \overline{Div}(t) + \overline{V}(t+1) - \overline{NF}(t)] \tag{4.2}$$

Now provided that the firm has established $\overline{I}(t)$ as the level of investment for period (t), the amount of new funds raised must satisfy

$$\overline{NF}(t) = \overline{I}(t) - [\overline{X}^{\tau}(t) - \overline{Int}(t) - \overline{Pfd}(t) - \overline{Div}(t)] \tag{4.3}$$

where $\overline{X}^{\tau}(t)$ represents expected earnings after taxes but before interest charge deductions and dividend payments, i.e.,

$$\begin{aligned}\overline{X}^{\tau}(t) &= [\overline{X}(t) - \overline{Int}(t)](1-\tau) + \overline{Int}(t)\\ &= (1-\tau)\overline{X}(t) + \tau \cdot \overline{Int}(t) \end{aligned} \tag{4.4}$$

where $\overline{X}(t)$ is expected earnings before interest and taxes. Substituting (4.3) into (4.1) yields

$$V(t) = \frac{\overline{X}^{\tau}(t) - \overline{I}(t) + \overline{V}(t+1)}{1+Z} \tag{4.5}$$

Using expression (4.5) to iteratively substitute for $\overline{V}(t+1)$, $\overline{V}(t+2)$, etc., and realizing that for V(t) to be finite then

$$\lim_{t\to\infty} \left|\frac{\overline{V}(t+1)}{(1+Z)^{t+1}}\right| = 0$$

results in a value for the firm at t = 0 of

$$V(o) = \sum_{t=1}^{\infty} \frac{\overline{X}^{\tau}(t) - \overline{I}(t)}{(1+Z)^{t}} \tag{4.6}$$

Equation (4.6) expresses the value of a firm as being equal to the present value of payments to holders of the firm's capital less any portion of current and future value apportioned to new holders of capital.

It should be emphasized that V(o) is the total market value of the firm and equal to the sum of the market values of the firm's component financing instruments--D(t), S(t), and F(t)--the total market values of the firm's debt, common stock, and preferred stock respectively.

Alternative Formulation of Equation (4.6)

Several formulae can be derived from (4.6) depending upon assumed characteristics of the firm's future earnings. First, specify investment in net additions to the firm's assets for period t as

$$\bar{I}(t) = \gamma \overline{X^{\tau}}(t) \qquad (4.7)$$

Expressing $\bar{I}(t)$ as a fraction of earnings follows M & M [38, equation (21)]. However, M & M and Arditti and Pinkerton [8] expressed investment for the unlevered firm as a fraction of income after tax

$$\bar{I}(t) = \alpha(1 - \tau)\bar{X}(t) \qquad (4.8)$$

Given that the level of investment each period is exogenously determined, it matters only for convenience how $\bar{I}(t)$ is defined. As can be seen by comparing (4.7) and (4.8)

$$\gamma = \frac{\alpha(1 - \tau)\bar{X}}{\bar{X}^{\tau}} \qquad (4.9)$$

Next is postulated an after-tax rate of return $r > Z$ expected to be earned on each period's net investment; the duration of $r > Z$ will be specified presently. Thus, at the end of period 1, or the beginning of period 2, investors expect total income to equal the income earned on existing assets, plus incremental income equal to $\gamma\overline{X}^\tau(o)\cdot r$ from the additional net investment. Thus

$$\overline{X}^\tau(1) = \overline{X}^\tau(o) + \gamma\overline{X}^\tau(o)\cdot r \tag{4.10}$$

$$\overline{X}^\tau(1) = \overline{X}^\tau(o)(1 + \gamma r) \tag{4.10a}$$

$$\overline{X}^\tau(2) = \overline{X}^\tau(o)(1 + \gamma r)^2 \tag{4.10b}$$

and for any period t

$$\overline{X}^\tau(t) = \overline{X}^\tau(o)(1 + \gamma r)^t \tag{4.10c}$$

It must be pointed out that r and ρ^* are not one in the same. Unlike the Arditti-Pinkerton model in which the incremental tax subsidy resulting from leveraged growth was separately accounted for (see equation (2.40)), expression (4.6) includes the incremental tax subsidy for period t (if it exists) in $\overline{X}^\tau(t)$. Thus, defining the rate of return to be earned on net additional investment in period t as

$$r(t) = \frac{\Delta\overline{X}^\tau(t)}{\overline{I}(t)} \tag{4.11}$$

is equivalent to

$$r(t) = \rho^* + \frac{\tau\Delta\text{Int}(t)}{\overline{I}(t)} \tag{4.12}$$

It will be recalled that ρ^* is the expected return net of taxes to be earned on net investment for the all equity firm.

Infinite horizon growth model. Specifying equation (4.6) in terms of (4.8) and (4.10) results in

$$V(o) = \overline{X}^T(o)(1 - \gamma) \sum_{t=1}^{\infty} \frac{(1 + \gamma r)^t}{(1 + Z)^t} \qquad (4.13)$$

Now for (4.13) to converge to a positive finite value requires that $0 \leq \gamma < 1$ and $\gamma r < Z$. If these conditions are met then

$$V(o) = \frac{\overline{X}^T(1)(1 - \gamma)}{Z - \gamma r} \qquad (4.14)$$

Anyone familiar with the constant growth model for a share of common stock (see equation (3.5)) will recognize the structure of (4.14). Furthermore, as with any growth mode, if $r = Z$, then

$$V(o) = \frac{\overline{X}^T(1)}{Z} \qquad (4.15)$$

Finite horizon growth model. For a variety of reasons--regulatory restraints is one--a finite horizon model may better describe expected income of the electric utility firm better than the constant growth model. Two derivations of the finite growth model follow. The first derivation restricts the number of periods in which net investment in additional assets takes place; the second derivation restricts the number of periods in which r is greater than Z.

Consider a firm which for $T - 1$ periods has expected net investment in additional assets of $\overline{I}(t) = \gamma\overline{X}^T(t)$. Subsequent to period $T - 1$ net investment each period is zero. On all additional net investment the firm is expected to earn a rate of return r greater than the cost of capital. Thus, the present value of the firm is

$$V(o) = \sum_{t=1}^{T-1} \frac{(1 - \gamma)\overline{X}^{\tau}(t)}{(1 + Z)^{t}} + \sum_{t=T}^{\infty} \frac{\overline{X}^{\tau}(t)}{(1 + Z)^{t}} \qquad (4.16)$$

As equations (4.10a, b, c) demonstrate, investing $\alpha\overline{X}^{\tau}$ at the end of period o or the begining of period 1, and earning r percent on that investment each year means that expected earnings at the end period t are

$$\overline{X}^{\tau}(t) = \overline{X}^{\tau}(o)(1 + \gamma r)^{t} \text{ for } t = o, 1, \ldots, T - 1 \qquad (4.17)$$

and

$$\overline{X}^{\tau}(t) = \overline{X}^{\tau}(o)(1 + \gamma r)^{T} \text{ for } t = T, T + 1, \ldots, \infty \qquad (4.18)$$

Substituting (4.17) and (4.18) into equation (4.16) results in

$$V(o) = (1 - \gamma)\overline{X}^{\tau}(o) \sum_{t=1}^{T-1} \frac{(1 + \gamma r)^{t}}{(1 + Z)^{t}} + \sum_{t=T}^{\infty} \frac{\overline{X}^{\tau}(o)(1 + \gamma r)^{T}}{(1 + Z)^{t}} \qquad (4.19)$$

Writing (4.19) in closed form yields

$$V(o) = \frac{(1 - \gamma)\overline{X}^{\tau}(1)}{Z - \gamma r} \left[1 - \frac{(1 + \gamma r)^{T}}{1 + Z} \right] + \frac{\overline{X}^{\tau}(o)(1 + \gamma r)^{T}}{Z(1 + Z)^{T}} \qquad (4.20)$$

To further simplify equation (4.20) assume $h \equiv \frac{1 + \gamma r}{1 + Z}$ is close to 1 and expand h^{T} about 1 dropping all but the first two terms of the Taylor series. This simplification results in

$$\left[\frac{1 + \gamma r}{1 + Z}\right]^{T} \approx 1 + T \left[\frac{\alpha r}{1 + Z}\right] \qquad (4.21)$$

Substitution of (4.21) into (4.20) results in

$$V(o) = \frac{\overline{X}^{\tau}(1)}{Z} + \frac{\gamma\overline{X}^{\tau}(1)}{Z(1 + Z)} [r - Z] T \qquad (4.22)$$

Equation (4.22) is a simplified expression for firm value. It is characterized by a finite number of future periods in which investment in additional assets is possible, but in those periods in which it does take place, a rate of return greater than the cost of capital will be earned forever.

The second derivation of equation (4.22) assumes that investment in additional assets will occur in every future period. However, the rate of return on investment is expected to be greater than the cost of capital for only a finite number of periods, t = 1, . . . , T. Subsequent to period T the rate of return on net investment is expected to equal the cost of capital. Thus, the following income streams will occur

$$\overline{X}^{\tau}(t) = \overline{X}^{\tau}(o)(1 + \gamma r)^{t} \text{ for } t = 0, 1, \ldots, T - 1 \tag{4.23}$$

and

$$\overline{X}^{\tau}(t) = \overline{X}^{\tau}(o)(1 + \gamma r)^{T}(1 + \gamma Z)^{t-T} \text{ for } t = T, T + 1, \ldots, \infty \tag{4.24}$$

When (4.7), (4.23) and (4.24) are substituted into equation (4.6), the general valuation model becomes

$$V(o) = \sum_{t=1}^{T-1} \frac{(1 - \gamma)\overline{X}^{\tau}(o)(1 + \gamma r)^{t}}{(1 + Z)^{t}} + \sum_{t=T}^{\infty} \frac{(1 - \gamma)\overline{X}^{\tau}(o)(1 + \gamma r)^{t}(1 + \gamma Z)^{t-T}}{(1 + Z)^{t}} \tag{4.25}$$

Expression (4.25) can be written in closed form as

$$V(o) = \frac{(1 - \gamma)\overline{X}^{\tau}(o)}{Z - \gamma r}\left[1 - \frac{(1 + \gamma r)^{T}}{1 + Z}\right] + \frac{\overline{X}^{\tau}(o)(1 + \gamma r)^{T}}{Z(1 + Z)^{T}} \tag{4.26}$$

Equation (4.26) is identical to equation (4.20) which it will be recalled was derived under the assumption that $I(t) = \gamma\overline{X}^{\tau}(t)$ prior to period T and zero thereafter. Thus, both variations of the finite growth model provide

$$V = \frac{\overline{X}^{\tau}}{Z} + \frac{\gamma\overline{X}^{\tau}}{Z(1 + Z)} [r - Z] \; T \qquad (4.27)$$

as an expression of total firm value where V and $\overline{X}^{\tau}$ replace V(o) and $\overline{X}^{\tau}(1)$ as total current market value and expected current tax-adjusted earnings, respectively.

An Examination of the Model

Although (4.27) is a highly simplified valuation model, all necessary elements of value are included for a firm of defined risk. The dependent variable V is the sum of the market values of all outstanding securities and thus reflects the level of each type of financing in the capital structure. Furthermore, V is specified as the sum of the present values of two income streams: a permanent component determined by the capitalization of expected earnings generated from existing capital assumed to last forever, and a growth component. The growth component is the approximate capitalized product of three elements: (1) anticipated investment in additional assets each period represented by $\gamma\overline{X}^{\tau}$; (2) the expected profitability of that investment indicated by the spread between r and Z; and (3) the duration of that profitability denoted by T. The average cost of capital Z, it should be noted, appears in both the no-growth and growth components of (4.27).

As explained in Chapter I, tax-adjusted earnings of electric utilities are determined in part by the product of an established rate base RB and allowed rate of return Z_a. To the extent that investors garner information from the pronouncements of utility commissions, RB and Z_a will be used to form a consensus estimate of r. This consensus need not be equal to Z_a, nor must it equal the rate of return existing on the current level of assets. However, to retain the simplicity of equation (4.27) in the subsequent analysis, it will be assumed that Z_a is equal to r, and further, that r applies to both existing and future assets thus defining tax-adjusted earnings as

$$\overline{X}^{\tau} \equiv r \cdot RB \qquad (4.28)$$

Substituting (4.28) into equation (4.27) provides a finite growth valuation model for electric utilities as

$$V = \frac{r \cdot RB}{Z} + \frac{\gamma r \cdot RB}{Z(1 + Z)} \; [r - Z] \; T \qquad (4.29)$$

Properties of the Model

Firm value can be expected to change whenever any of the variables on the right hand side of equation (4.29) change. Assuming that over reasonable periods of time the rate base is constant, the total change in V is

$$dV = f^{T}dT + f^{r}d\overline{r} + f^{\gamma}d\gamma + f^{Z}dZ \qquad (4.3)$$

where f^{i} is the partial derivative of V with respect to the ith variable and di is the differential of variable i. The expression dV represents the sum of all changes from the four independent variables in (4.29). It is possible, of course, that V can change as a result of a change in only one of the independent variables. For example, if only the rate of return on investment changed, i.e., dT, dα, and dZ equal zero, then

$$f^r = \frac{dV}{dr} \; T,\ \gamma,\ Z \text{ constant} = \frac{dV}{dr} \tag{4.31}$$

In order to better understand equation (4.29), the impact on firm value of each independent variable will be analyzed, first by restricting other independent variables to current levels, and then by allowing for simultaneous changes in two independent variables. Also, a proposed regulatory reaction to a change in the average cost of capital will be explained.

Length of investing period, T. The greater the number of periods in which profitable investments occur, or equivalently, the longer an investment enjoys a rate of return greater than the cost of capital, the greater will be the value of the firm. Specifically, the change in value for a positive change in T is

$$f^T = \frac{\gamma r \cdot RB}{Z(1 + Z)} [r - Z] > 0 \tag{4.32}$$

which is the slope of the linear relationship in Figure 4.1.

Level of investment, γ. Since γ is current investment expressed as a fraction of current earnings, then as γ increases, with r > Z for a given duration T, it follows that the value of the firm will increase by

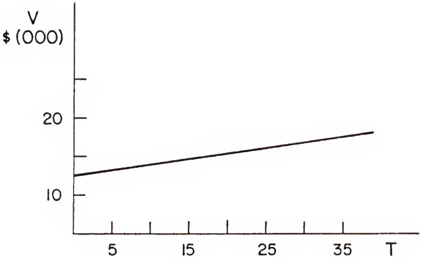

Figure 4.1

Value as explained by length of investment period when $Z = .08$, $r = .10$, $\gamma = 0.6$, and $RB = \$10,000$

$$f^{\gamma} = \frac{r \cdot RB}{Z(1 + Z)} [r - Z] T > 0 \qquad (4.33)$$

times the change in γ. Figure 4.2 illustrates the relationship graphically.

Note that equation (4.29) assumes a constant level of γ over T periods in the future. As pointed out above, the infinite horizon model, (see equation (4.14)) requires that γ be less than 1.0. Historical values of γ calculated for the sample for 20 years since 1955, indicated annual net investment to be on average greater than tax-adjusted earnings.

Expected tax-adjusted rate of return, r. The larger the expected after-tax return on investment, the greater will be the firm's present value. Partially differentiating equation (4.29) with respect to r gives an increasing function in r, expressed as

$$f^{r} = \frac{RB}{Z} \left[1 + \frac{\gamma T}{1 + Z} (2r - Z) \right] > 0 \qquad (4.34)$$

Figure 4.3 illustrates the relationship between firm value and the after tax rate of return on the rate base. The figure is a bit misleading. In the first place, it is unlikely that investors would expect a regulatory body to allow a utility to earn a rate of return exceeding its cost of capital to the degree and length of time depicted by the diagram. Secondly, even for a firm not encumbered by regulated return, the opportunity for locating investments which promise such excess returns without a corresponding increase in the business risk of the firm, and hence, a higher cost of capital, is rare.

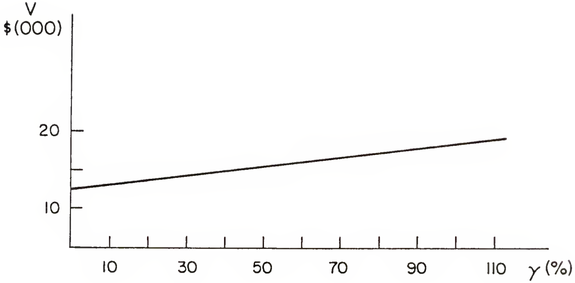

Figure 4.2

Value with respect to investment as a fraction of earnings when
Z = .08, r = .10, T = 25, and RB = $10,000

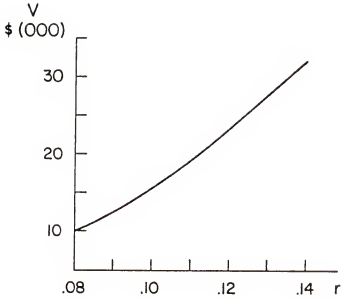

Figure 4.3

Value as explained by expected after tax rate of return on investment when $Z = .08$, $\alpha = .60$, $T = 25$, and $RB = \$10,000$

Changes in the average cost of capital, Z. Change in total firm value V resulting from changes in γ, and T entered only through the growth term of equation (4.29), the perpetual income component being unaffected. Assuming that the average return on the existing rate base and the marginal return to be earned on additions to that rate base were equal, the tax-adjusted rate of return r determined the levels of both the permanent and growth components of total value. The cost of capital also affects the valuation equation through both the perpetual and growth components of the model. In the nongrowth component the present value of the perpetual expected income stream varies inversely with the cost of capital. In the growth component the approximate level of growth income is established in part by the cost of capital and the resulting income stream is divided by Z(1 + Z) to calculate its approximate present value.

To see the total impact on firm value from a change in the average cost of capital, partially differentiate equation (4.29) with respect to Z obtaining

$$f^Z = -\frac{r \cdot RB}{Z^2}\left[1 + \frac{\gamma T}{(1+Z)^2}(r + 2rZ - Z^2)\right] < 0 \qquad (4.35)$$

It should be emphasized that the change described by inequality (4.35) decreases as the average cost of capital increases. This diminishing impact upon firm value can be identified by taking the second partial derivative with respect to Z of (4.29) giving

$$\frac{\partial^2 V}{\partial Z^2} = \frac{2r \cdot RB}{Z^3}\left[\frac{\gamma T}{(1+Z)^3}(r + 3rZ + 3rZ^2 - Z^3)\right] > 0 \qquad (4.36)$$

Figure 4.4 shows constructed values of four hypothetical companies, each with a rate base currently valued at \$10,000; annual investment in income producing assets equal to 60 percent of that year's earnings; and each year's investment having an average life of 25 years. Only r, the after-tax expected rate of return varies among the four. Firm 1 is defined as a no-growth company, i.e., $r_1 = Z$. Firm 2 differs from Firm 1 in that the net rate of return on investment is assumed to be 14 percent regardless of the level of Z. Finally, Firms 3 and 4 illustrate a constant spread of 200 and 300 basis points respectively between investors' consensus return on investment and the average cost of capital. It should be noted that in spite of this constant spread (e.g., if Z increases from .10 to .11 for Firm 3, then r_3 instantly increases from .12 to .13) firm value declines as Z increases. It will now be shown that for $r > Z$, investors must expect the change in r to be greater than the change in Z for firm value not to decrease as Z increases.

Regulating the allowed rate of return. The following scenario provides an illustration of two variables changing simultaneously. Recall that r represents investors' consensus expected rate of return on all assets of the firm, both average and marginal. Now allow interest rates to increase to the extent that the utility's cost of capital increases by 100 basis points from 10 to 11 percent. Expression (4.35) and Figure 4.4 show that, regardless of the size of r relative to Z, the value of the utility will decline as Z increases. The task of the regulatory agency inclined

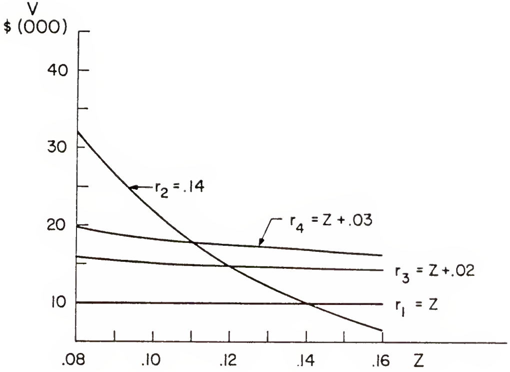

Figure 4.4

Value with respect to average cost of capital when $\gamma = .60$, $T = 25$, and $RB = \$10,000$

to provide rate relief is to determine what rate of return the utility should now be permitted if the value of the firm is to remain unchanged.

Assuming that the values investors assign to γ and T remain constant and, furthermore, that investors believe $r = Z_a$, the change in firm value can be determined as follows. First, express the total change in V from a change in Z and r as the differential

$$dV = \frac{-r \cdot RB}{Z^2} \left[1 + \frac{\gamma T}{(1 + Z)^2} (r + 2rZ - Z^2) \right] dZ$$
$$+ RB \left[\frac{1}{Z} + \frac{2\gamma rT}{Z(1 + Z)} + \frac{\gamma T}{1 + Z} \right] dr \qquad (4.37)$$

A regulatory objective of maintaining current firm value is equivalent to setting dV = 0; the change in r associated with dV = 0 is

$$dr = \left[\frac{(1 + Z)^2 + \gamma T (r + 2rZ - Z^2)}{(1 + Z)^2 + \gamma T [r + 2rZ - Z^2 + (r - Z)]} \right] \frac{dZ}{Z} r \qquad (4.38)$$

To simplify (4.38) substitute the identity

$$f(r,Z) \equiv \frac{(1 + Z)^2 + \gamma T (r + 2rZ - Z^2)}{(1 + Z)^2 + \gamma T [r + 2rZ - Z^2 + (r - Z)]} \qquad (4.39)$$

for the bracketed term in (4.38) and note that f(r,Z) is less than, equal to, or greater than 1.0 when r is greater than, equal to, or less than Z. Rewriting (4.38) in this simplified form gives

$$dr = f(r,Z) \frac{dZ}{Z} r \qquad (4.40)$$

or

$$\frac{dr}{r} = f(r,Z) \frac{dZ}{Z} \qquad (4.41)$$

Equation (4.40) provides a specific model to regulate a public utility's rate of return on its rate base. To implement this idea, the utility commission must obtain current estimates of Z, r, γ, and T and be content with allowing a rate of return which would not change the total value of the firm. As an example of setting a new allowed return, consider the case of Firm 3 described above and in Figure 4.4. Recall that for Firm 3 the expected rate of return was constructed to be $r = Z + .02$. Thus, at $Z = .10$, $r = .12$ and $V = \$15,273$. When the cost of capital increased by 100 basis points to $Z = .11$, the rate of return on assets (rate base) increased to $r = Z + .02 = .13$, but firm value descreased to $V = \$15,012$. The reason for the decline, of course, is that r and Z increased by equal absolute amounts. To find that Z_a which leaves the value of the firm unchanged, equations (4.39) and (4.41) can be used. First solving (4.39)

$$f(.12,\ .10) = 0.914773$$

Now using (4.41) it is seen that the percentage change in r must be

$$\frac{dr}{r} = .914773\left(\frac{.01}{.10}\right) = .091477$$

or the new allowed rate of return must be

$$Z_a = (1.091477)\ 0.12 = 0.131$$

to ensure that firm value remains at $V = \$15,273$ (approximately).

Estimating the Average Cost of Capital

There are two ways that equation (4.22) can be used to estimate the average cost of capital for a firm. The first way is straightforward and appropriate for an individual firm under conditions of market equilibrium. If numerical values are obtained for V, $\overline{X}^\tau$, γ, r and T, they can be substituted into (4.22) to obtain a corresponding Z value. The second procedure is more general. If it is assumed that Z and T are identical for all firms in a given sample of equal risk firms, then numerical estimates for them can be obtained by regressing on the appropriate variables in the equation. It is the second method on which attention is now directed. In this section a statistical equation is constructed for equation (4.22) that is nonlinear in its parameters. An iterative least-squares algorithm is then proposed to calculate maximum likelihood estimates of the average cost of capital. Numerical results of cross-section regressions are presented in Chapter V.

The Structural Model

To construct the structural model the growth term of expression (4.22) is first expanded to

$$V = \frac{1}{Z}\overline{X}^\tau - \frac{T}{1 + Z}\gamma\overline{X}^\tau + \frac{T}{Z(1 + Z)}\gamma\overline{X}^\tau r \tag{4.42}$$

A straight forward regression equation corresponding to (4.42) is

$$V = \beta_1 G_1 + \beta_2 G_2 - \beta_1\beta_2 G_3 + u \tag{4.43}$$

where

$$\beta_1 = \frac{1}{Z}$$

$$\beta_2 = \frac{T}{1 + Z}$$

$$G_1 = \overline{X}^{\tau}$$

$$G_2 = \gamma\overline{X}^{\tau}$$

$$G_3 = \gamma\overline{X}^{\tau}r$$

Note that the coefficient associated with independent variable G_3 is restricted in equation (4.43) to the negative product of the coefficients of G_1 and G_2. As a result of this nonlinear restriction equation (4.43) is overidentified, i.e., there is no unique solution to the restricted parameter that corresponds to independent variable G_3. However, it is still possible to estimate the restricted parameters of (4.43) using a nonlinear approach which minimizes the error sum of squares

$$S(\beta_i\beta_2) = \sum_{i=1}^{n} \left[v_i - \beta_1 G_{1i} - \beta_2 G_{2i} + \beta_1\beta_2 G_{3i} \right]^2 \tag{4.44}$$

Estimation of β_1 and β_2

Several nonlinear algorithms are available for estimating the coefficients that minimize $S(\beta_1,\beta_2)$. The one employed here is the modified Gauss-Newton method, also called the linearization (by truncated Taylor series) method.[3] To use the linearization method, expand equation (4.43) about initial estimates of the coefficients, b_{10} and b_{20}, curtailing the expansion at the first derivitive. This operation results in

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[3] For a brief overview of nonlinear estimation in general and the linearization method in particular see Draper and Smith [18].

$$V = [b_{10}G_1 + b_{20}G_2 - b_{10}b_{20}G_3] + [G_1 - b_{20}G_3](\beta_1 - b_{10})$$
$$- [G_2 - b_{10}G_3](\beta_2 - b_{20}) + u_o \tag{4.45}$$

The zero in each subscript indicates the particular iteration of the algorithm; b_{10} is the initial estimate of β_1, b_{11} is the estimate of β_1 after one iteration, b_{12} the estimate of β_1 after the second iteration and so on.

Now equation (4.45) can be reformed as

$$V - f^o(\underline{G};\, b_{10},b_{20}) = [G_1 + b_{20}G_3]\ A_{10}$$
$$- [G_2 - b_{10}G_3]\ A_{20} + u_o \tag{4.46}$$

where

$$f^o(\underline{G};\, b_1,b_2) = b_{10}G_1 + b_{20}G_2 - b_{10}b_{20}G_3 \tag{4.46a}$$
$$A_{10} = \beta_1 - b_{10} \tag{4.46b}$$
$$A_{20} = \beta_2 - b_{20} \tag{4.46c}$$

Note that (4.46a) is linear in its coefficients. Thus, ordinary least-squares regression can be used to calculate estimates a_{10} and a_{20} for coefficients A_{10} and A_{20}. From equations (4.46b) and (4.46c) can be calculated new estimates of β_1 and β_2 as

$$b_{11} = b_{10} + a_{10} \tag{4.46b'}$$

and

$$b_{21} = b_{20} + a_{20} \tag{4.46c'}$$

Coefficients b_{11} and b_{22} are then substituted into equation (4.45) in place of the initial estimates b_{10} and b_{20} and a new linear estimating equation is formed as

$$V - f^1(\underline{G};\, b_{11},b_{21}) = G_1 + b_{21}G_3)A_{11} - [G_2 - b_{11}G_3]A_{21} + u_1 \qquad (4.46')$$

Parameters A_{11} and A_{21} are once again estimated by ordinary least-squares regression and estimates for β_1 and β_2 are revised as before, i.e.,

$$b_{12} = b_{11} + a_{11} \qquad (4.46b'')$$

$$b_{22} = b_{21} + a_{21} \qquad (4.46c'')$$

The iterations continue until

$$\frac{b_{1,\, j+1} - b_{1j}}{b_{1j}} < \delta \qquad (4.47a)$$

and

$$\frac{b_{2,\, j+1} - b_{2j}}{b_{2j}} < \delta \qquad (4.47b)$$

where δ is a prespecified convergence criterion.

Statistical Properties of the Model

Estimates obtained by iterative least-squares are exactly the same as maximum likelihood estimates provided that the maximization of the likelihood function is achieved by minimizing the sum of squared deviations between the actual and fitted values of the dependent variable. The maximum likelihood estimates thus obtained are unbiased, consistent and asymptotically efficient provided the following conditions hold:

a. u_i is normally distributed

b. $E(u_i) = 0$

c. $E(u_i^2) = \sigma^2$

d. $E(u_i, u_j) = 0, i \neq j$

e. $\underline{G}$ and $\underline{u}$ are independent

f. A global maximum is reached

Sample data should, of course, be examined to determine if these conditions are satisfied. However, some of the conditions are less crucial than others. For instance, it can be shown that the asymptotic distribution of nonlinear estimates is normal with identical mean and variance as the asymptotic distribution of maximum likelihood estimates. Thus, the assumption of normality is not crucial in order to ensure desirable asymptotic properties. Also, of some importance, is the condition that $\underline{G}$ and $\underline{u}$ are independent since independence of the stocastic independent variables and the disturbance term ensures desirable properties of the estimators. However, if they are at least contemporaneously uncorrelated, the desirable properties of the estimators will still exist asymptotically.

Summary

This chapter has developed a theoretical model that expresses the total value of a firm. It was shown how the model could be utilized in the regulatory process. Also explained was a method for calculating maximum likelihood estimates of the cost of capital for a sample of firms of equal risk. The results of cross-section regression of equation (4.43) will now be presented in Chapter V.

CHAPTER V

THE RESULTS

This chapter presents evidence confirming the major hypothesis that the cost of capital can be estimated directly by the valuation model

$$V = \frac{1}{Z}\,\overline{X}^{\tau} - \frac{T}{(1 + Z)}\,\gamma\overline{X}^{\tau} + \frac{T}{Z(1 + Z)}\,\gamma\overline{X}^{\tau}\,r \qquad (4.42)$$

where

V = the total market value of the firm

$\overline{X}^{\tau}$ = expected future level of total tax-adjusted earnings for the firm

Z = the average cost of capital

T = the number of years of anticipated growth in tax-adjusted earnings

γ = expected addition to net assets as a percentage of current tax-adjusted earnings

r = expected tax-adjusted rate of return on additional assets

In the process the following minor hypothesis is tested: Financial leverage does not effect the total value of an electric utility. A test of the homogeneity of electric utilities with respect to business risk is reported in Appendix B.

Chapter V is organized as follows. Section one identifies the model's variables and describes the data associated with these variables. The second section of the chpater reports the results of cross-section regressions for the entire sample for years 1963, 1965, 1967, 1969, 1971, and 1973. In section three the minor hypothesis is tested.

Defining and Measuring the Variables

To test the major and minor hypotheses seventy Class A and B electric utilities included in the Compustat utility file were initially identified; one was subsequently discarded because of incorrect data. The remaining sixty-nine firms are listed in Appendix A. From equation (4.42) the following statistical equation is proposed

$$V_i = \beta_1 \overline{X}^{\tau}{}_i + \beta_2 \overline{I}_i + \beta_1 \beta_2 \Delta \overline{X}^{\tau}{}_i + u_i \qquad (5.1)$$

where

V_i = total market value of firm i

$\overline{X}^{\tau}{}_i$ = total tax-adjusted earnings expected to be earned on the existing level of assets by firm i

$\overline{I}_i$ = expected net investment in additional assets by firm i

$\Delta\overline{X}^{\tau}{}_i$ = expected change in total tax-adjusted earnings as a result of additional asset investment b, firm i

β_1 = 1/Z, the marginal capitalization rate for the electric utilities

β_2 =-T/(1 + Z) where T is the expected number of years for which $r > Z$

u_i = a random disturbance term for firm i.

The variables in (5.1) are not easily obtained. Those on the right-hand side of the equation are unobservable; they are best described as a consensus of investors' expectations about the future prospects of the firm. Total firm value, the dependent variable, is observable except for those firms which have privately placed bond and preferred stock issues. There being no market prices for these securities, one can either include them at their recorded value, a distasteful choice in periods of changing interest rates, or the alternative, estimate their market value and use the estimate in calculating the total market value of that firm's securities.

The Dependent Variable

Total firm value is defined as the sum of all outstanding financial obligations specifically identified with either an owner or creditor. Thus,

$$V \equiv \sum_{j=1}^{Md} Pb_j \cdot Nb_j + \sum_{j=1}^{Md} Pp_j \cdot Np_j + Pc \cdot Nc + Po \cdot No$$

where

Pb_j = market price per bond in bond series j

Pp_j = market price per share of preferred stock in preferred stock series j

Po = market price per share of common stock

Pc = market price of a unit of "other financing"

Nb_j = number of bonds in bond series j

Np_j = number of shares of preferred stock in preferred stock series j

Nc = number of shares of common stock

N_O = number of units of "other financing"

Md = number of bond series for a given firm

Mp = number of preferred stock series for a given firm.

Other financing. Bank loans, accruals, and miscellaneous short-term credit is lumped together and called "other financing." It has no observable market value and so, for lack of an alternative procedure, it is valued as the average of the beginning and ending balances for the year. If interest rates rise during a particular year, the procedure has the effect of overstating the true value of "other financing." However, the short maturity of "other financing" tends to make the overstatement quite small. Furthermore, "other financing" relative to total firm value is also small, reducing further the error resulting from the use of recorded values for this component.

Common stock. To calculate the market value of each firm's common equity the average of the high and low closing stock price for the year is multiplied by the average number of shares outstanding during the year.

Bond values. The market value of each firm's outstanding bonds is found by summing the market values of each bond series. The average of the annual high and low prices of each bond series, as reported in Moody's Utility Manual, times the number of bonds outstanding during that year was used as the market value of a particular series. However, as noted above, some bond issues were privately placed. Not being traded, the market prices of these bonds had to be estimated.[1]

Several statistical equations were examined for estimating the market value of non-traded bonds. To select the best model, the entire sample of bonds with observed average prices was divided into two groups. Equation coefficients were estimated with one group, and fitted prices for the second group were calculated using these estimated coefficients. The equation selected, based upon the smallest mean square prediction error between fitted and actual prices, was the linear equation

$$P_b = \alpha_o + \alpha_1 M + \alpha_2 C + \alpha_3 D_1 + \alpha_4 D_2 + \alpha_5 D_3 + u \qquad (5.3)$$

where

M = the maturity of the bond in months

C = the coupon rate of the bond

D_1 = a dummy variable equal to 1 if the bond is rated Aa and 0 for other ratings

D_2 = a dummy variable equal to 1 if the bond is rated A and 0 for other ratings

D_3 = a dummy variable equal to 1 if the bond is rated Baa or less and 0 for other ratings.

Equation (5.3) is not to be viewed as a pricing model for corporate debt. Rather, it is the best in terms of minimum mean square prediction error of several ad hoc procedures available for estimating prices of non-traded bonds.

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[1] The greatest amount of privately placed debt outstanding was in 1973 when it comprised 19 percent of the total funded debt.

Preferred stock. The market value of each firm's preferred stock was calculated identically to bond market values, i.e., the average of the high and low annual price was multiplied by the number of preferred stock shares for the traded issues. For non-traded issues the market price was estimated by the linear equation

$$P_p = \alpha_o + \alpha_1 D_p + u \tag{5.4}$$

where D_p represents the preferred stock dividend. As with bond price estimation, coefficients of (5.4) were estimated from the sample of preferred stock issues with observed prices, and then used to fit the unobserved preferred stock prices. Of the price prediction equations examined, including the use of dummy variables to reflect preferred stock ratings, equation (5.4) was superior to all others in terms of minimum mean square prediction error.

Firm total value. Each firm's total value was calculated according to equation (5.2). Summary statistics of total and component values appear in Table 5.1. The numbers reveal that total market values of electric utilities increased on average by more than 50 percent during the ten-year sample period. The spread between recorded market values became larger during this period for the obvious reason that yields on debt and preferred stock increased. The inclusion of debt and preferred stock at book value would thus overstate firm total market value; for example, in 1973 the

Table 5.1

Summary of Total Firm Value for 69 Electric Utilities ($ millions)

Year	Firm Market Value	Common Stock Value	Funded Debt Value		Preferred Stock Value		Other Financing
			Market	Book	Market	Book	
1963							
Mean	694.94	419.07	222.66	238.26	48.09	47.08	5.12
S.D.	700.54	406.25	255.34	273.34	66.60	65.20	9.32
Min	98.80	63.92	28.91	30.94	0.0	0.0	0.0
Max	3461.83	1999.90	1388.64	1485.97	409.89	401.30	46.75
1965							
Mean	783.83	489.16	237.92	260.06	47.08	48.97	7.67
S.D.	785.52	473.62	276.74	302.49	65.93	68.58	10.59
Min	144.70	93.02	33.20	36.29	0.0	0.0	0.0
Max	3912.70	2282.38	1565.37	1710.97	427.82	445.00	53.78
1967							
Mean	748.78	423.94	263.42	313.34	47.96	58.99	13.45
S.D.	763.24	424.25	296.87	350.87	64.22	78.99	16.65
Min	137.84	73.45	30.45	35.98	0.0	0.0	0.0
Max	1824.66	2084.97	1558.23	1841.64	434.95	515.00	77.5
1969							
Mean	771.20	404.82	288.11	376.46	48.81	68.95	29.46
S.D.	763.48	396.56	311.41	409.52	64.11	90.55	28.17
Min	141.63	74.33	38.70	50.58	0.0	0.0	0.0
Max	3798.35	2026.42	1516.55	1981.64	441.59	626.54	136.08
1971							
Mean	920.88	413.89	404.14	481.35	76.46	98.79	26.18
S.D.	900.91	392.24	425.97	507.35	92.24	119.37	29.46
Min	158.69	75.84	55.74	66.38	0.0	0.0	0.0
Max	4218.00	1977.66	2021.87	2408.13	523.34	676.15	140.0
1973							
Mean	1051.38	404.53	491.44	574.05	109.91	118.77	45.49
S.D.	1004.45	370.68	500.62	584.77	121.44	353.34	45.76
Min	158.09	67.05	56.71	66.24	0.0	0.0	0.0
Max	4561.16	1763.16	2433.88	2852.90	594.72	750.91	220.15

overstatement would be 11 percent. Additional insight into the relative importance of each component of financing can be gained from Table 5.2 where it is observed that the increase in firm total value resulted primarily from substantial increases in the use of long-term debt and preferred stock. Also, the large increase in short-term financing should not go unnoticed. While far from being a major component of firm value (about 4 percent in 1973), it is evident that utilities have increasingly relied upon short-term methods to finance assets.

The Independent Variables

As noted above, none of the independent variables of equation (5.1) are observable. Rather, they represent investors' consensus judgment about the future prospects of each firm's earnings and investments. Before explaining a method for generating naive[2] expected values for the independent variables, a definition of each is provided.

Tax-adjusted earnings.

Tax-adjusted earnings are earnings generated from the current level of net assets and assumed to continue ad infinitum. Annual tax-adjusted earnings are taken from the income statement as the sum of component incomes to debtholders, preferred stockholders and common stockholders

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[2] Naive in the sense that only historical data is employed to predict future levels of a variable.

Table 5.2

Component Market Values as a Percent of Firm Total Market Value

Year	Common Stock	Funded Debt	Preferred Stock	Other Financing	Firm Value
1963	.604	.320	.069	.007	1.000
1965	.626	.304	.060	.010	1.000
1967	.566	.352	.064	.018	1.000
1969	.525	.374	.063	.038	1.000
1971	.449	.439	.083	.029	1.000
1973	.385	.467	.105	.043	1.000

$$X^{T}(t) = X(t) - Tx(t) \qquad (5.5)$$

where

$X(t)$ = total earnings before interest and taxes

$Tx(t)$ = total taxes paid in period t

Equation (5.5) assumes that earnings available to owners of common stock are comparable for companies with different depreciation policies, e.g., normalized versus flow-through; unlike McDonald [36, n. 12], no adjustment was made.[3]

Net investment in additional assets. The annual change in the recorded value of depreciated assets represents the annual net investment in total assets. Thus, net investment reflects additions to plant and equipment as well as increases in working capital. Annual net investment for each firm was calculated as

$$I(t) = A(t) - A(t - 1) \qquad (5.6)$$

where

$A(t)$ = total assets in period t.

Incremental earnings from additional net investment. Annual incremental earnings represent the annual change in total tax-adjusted earnings and are calculated as

$$\Delta X^{T}(t) = X^{T}(t) - X^{T}(t - 1) \qquad (5.7)$$

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[3] For additional information on this point see Brigham and Pappas [14].

Forecasts of Independent Variables

It is not possible to correctly measure the ex ante values of the independent variables. However, it is possible to generate from a firm's existing data, numbers to serve as estimates of investors' consensus values for ex ante earnings and investments. Although other forecasting techniques exist, the four procedures discussed below have frequently been cited in other valuation studies including those reviewed in Chapter III.

Regression. "Normalizing" a time series of economic data has long been employed by security analysts to predict future levels of variables. One straightforward procedure is to calculate by ordinary least squares a linear or exponential trend of the particular time series, then use the trend line to predict future values of the series.

Four-year average. Miller and Modigliani [39] estimated future investment in total assets as a four-year arithmetic average of annual changes in assets. As justification, M & M cited higher correlations with firm value for this procedure than for other estimating techniques.

Geometric mean of first ratios. Craig and Malkiel [15] examined forty different quantitative measures for estimating future earnings. Based on Thiel's Inequality Coefficient, the 10-year geometric mean was judged superior.

Distributed lag. McDonald [36] used a distributed lag over the previous 3 years with weights of .60, .24, and .16 to forecast changes in both earnings and investment in fixed assets.

Analysis of Prediction Methods

A total of 20 years of data was drawn from the Compustat Utility File to calculate one-year forecasts of tax-adjusted earnings and investment in total assets by each of the above forecasting methods. Depending upon the required number of observations, the number of one-year forecasts range from 16 per firm for the distributed lag model to 9 per firm for the 10-year trend and average. Actual and predicted values of tax-adjusted earnings and net investment were compared by Thiel's Inequality Coefficient

$$U^2 = \frac{\sum_{i=1}^{N} (Pre_i - Act_i)^2}{\sum_{i=1}^{N} Act_i^2} \tag{5.8}$$

where

N = the number of observations

Pre_i = the predicted change in the forecasted variable

Act_i = the actual change in the forecasted variable.

Tables 5.3 and 5.4 report the accuracy of the several predicting techniques described above. The 3-year distributed lag and the 5-year geometric mean consistently provide more accurate forecasts than any of the other methods. The worst by far is the 10-year linear trend. Because of its marginal superiority in predicting tax-adjusted earnings (the net investment predictions were

Table 5.3

The Quality of Several Investment Forecasts

No.	Method of Forecast	U
1	3-Year Distributed Lag	0.0250
2	4-Year Arithmetic Mean	0.0295
3	5-Year Geometric Mean (first ratio)	0.0250
4	10-Year Geometric Mean (first ratio)	0.0310
5	5-Year Linear Trend	0.0360
6	10-Year Linear Trend	0.0740
7	5-Year Exponential Trend	0.0310
8	10-Year Exponential Trend	0.0585

Table 5.4

The Quality of Several Tax-adjusted Earnings Forecasts

No.	Method of Forecast	U
1	3-Year Distributed Lag	0.0390
2	4-Year Arithmetic Mean	0.0415
3	5-Year Geometric Mean (first ratio)	0.0375
4	10-Year Geometric Mean (first ratio)	0.0425
5	5-Year Linear Trend	0.0480
6	10-Year Linear Trend	0.0830
7	5-Year Exponential Trend	0.0420
8	10-Year Exponential Trend	0.0640

equally precise), the 5-year geometric mean was selected as the best method for generating investor consensus estimates of future tax-adjusted earnings and net investment. Specifically, each independent variable of equation (4.42) is estimated as

$$\overline{X}^{\tau}(t) = \left[X^{\tau}(t - 1)/X^{\tau}(t - 6) \right]^{1/5} \cdot X^{\tau}(t - 1) \qquad (5.9a)$$

$$\Delta\overline{X}^{\tau}(t) = \overline{X}^{\tau}(t) - X^{\tau}(t - 1) \qquad (5.9b)$$

$$\overline{I}(t) = \left[I(t - 1)/I(t - 6) \right]^{1/5} \cdot I(t - 1) \qquad (5.9c)$$

Heteroscedasticity

It is evident from Table 5.1 that large disparities in firm size exist. In 1963, for example, the maximum firm value was more than 35 times larger than the minimum. It follows, a priori, that disturbances about the regression surface will be positively correlated with firm size resulting in inefficient estimated coefficients. One approach to correct non-constant variance is logarithmic or exponential transformation of the variables in equation (5.1). However, since the variables of (5.1) are additive, such a transformation introduces a specification bias.

A superior method of data adjustment is to weight each observation inversely to the size of its respective error term. For equation (5.1) an obvious candidate is V, total firm value. Dividing (5.1) by V results in

$$\frac{\overline{X}^{\tau}}{V} = \alpha_o + \alpha_1 \frac{\overline{I}}{V} + \alpha_2 \frac{\Delta\overline{X}^{\tau}}{V} + e \qquad (5.10a)$$

where

$\alpha_o = 1/\beta_1$

$\alpha_1 = -\beta_2/\beta_1$

$\alpha_2 = -\beta_2$

$e = u/V$

Although (5.10a) appears to assure homoscedasticity, it potentially creates even more serious problems. First, V is stochastic and neither independent nor contemporaneously uncorrelated with u; thus the least squares estimators of (5.10a) will not be consistent. Measurement errors in V pose an additional problem (recall that some bond and preferred stock values had to be estimated). To the extent that measurement errors exist in V, the coefficients for investment and change in earnings will be asymptotically biased toward zero. Identical problems result if (5.1) is deflated by $\overline{X}^{\tau}$ giving

$$\frac{V}{\overline{X}^{\tau}} = \beta_1 + \beta_2 \frac{\overline{I}}{\overline{X}^{\tau}} + \beta_1\beta_2 \frac{\Delta\overline{X}^{\tau}}{\overline{X}^{\tau}} + w \qquad (5.10b)$$

Specific attention will be directed at measurement errors in $\overline{X}^{\tau}$ in a subsequent section.

Following previous cross-section econometric studies (cf. [39], [32], and [36] reviewed in Chapter III), the inverse of the book value of total assets was selected as the best possible weight for deflating the variables in equation (5.1). A priori, the strong relationship existing between the book value of a utility's total assets and its rate base results in positive correlation across firms between reported tax-adjusted earnings and total assets.

Furthermore, total assets can be considered as nonstochastic, at least considerably less for utilities than other industries due to the generally followed system of accounts required by the Federal Power Commission. Thus, the adjusted statistical model selected for use in the cross-section regressions is

$$\frac{V}{A} = \beta_1 \frac{\overline{X}^\tau}{A} + \beta_2 \frac{\overline{I}}{A} + \beta_1\beta_2 \frac{\Delta\overline{X}^\tau}{A} + u \tag{5.11}$$

Table 5.5 lists summary statistics of the dependent and independent variables, deflated by total assets. Fixed charge coverage, to be used in subsequent specification tests, is also included in Table 5.5. Table 5.6 reports their product correlations.

Omission of the Constant Term

Equation (5.11) does not have an intercept term. As developed in the theoretical valuation model, firm total value depends solely upon the tax-adjusted income expected by the owners and creditors of the firm and capitalized at an appropriate rate Z. Including a constant in the regression equation when prior knowledge requires that it be restricted to zero leads to more imprecise estimators than would be obtained if the intercept was constrained to zero. Nevertheless, it is a good idea to empirically verify a zero intercept, and this is undertaken below.

Results of the Nonlinear Regression

In this section the results of fitting the proposed valuation model by nonlinear least squares with the entire sample of 69 electric utilities are presented.

Table 5.5

Summary Statistics of Independent Variables Deflated by Total Assets

Statistics	Value	Expected Total Earnings[a]	Change in Total Earnings[b]	Expected Investment in Total Assets[c]	Fixed Charge Coverage[d]
1963					
Mean	1.36440	.06321	.00515	.06615	.01514
S.D.	.20014	.00574	.00175	.03043	.01169
Min.	1.07668	.05186	.00126	.01526	.00073
Max.	2.17303	.07721	.01026	.16461	.05743
1965					
Mean	1.39746	.06449	.00448	.05339	.01440
S.D.	.21633	.00674	.00191	.03150	.01108
Min.	1.08137	.05202	.00084	.01340	.00064
Max.	2.35064	.08548	.01269	.21112	.06121
1967					
Mean	1.13764	.06481	.00457	.04869	.01164
S.D.	.13436	.07065	.00141	.02211	.00932
Min.	.88956	.05076	.00157	.01249	.00056
Max.	1.57163	.08129	.00918	.14930	.05266
1969					
Mean	.98549	.06245	.00448	.06181	.00842
S.D.	.11218	.00586	.00143	.01982	.00712
Min.	.78935	.05192	.00108	.01988	.00046
Max.	1.26140	.07850	.00893	.10754	.03789
1971					
Mean	.93975	.06473	.00562	.08111	.00553
S.D.	.09983	.00752	.00153	.02402	.00496
Min.	.78330	.04831	.00258	.03606	.00034
Max.	1.31726	.08279	.00925	.14908	.02884
1973					
Mean	.84682	.06741	.00716	.09628	.00406
S.D.	.06599	.00674	.00222	.03035	.00281
Min.	.72340	.05092	.00179	.02577	.00028
Max.	1.04826	.08432	.01350	.17579	.02453

[a] $\overline{X}^T(t)/A(t) = \{[X^T(t - 1)/X^T(t - 6)]^{1/5}\} \cdot X^T(t - 1)/A(t)$

[b] $\Delta\overline{X}^T(t)/A(t) = [\overline{X}^T(t) - X^T(t - 1)]/A(t)$

[c] $\overline{I}/A(t) = \{[I(t - 1)/I(t - 6)]^{1/5}\} \cdot I(t - 1)/A(t)$

[d] $FCC(t)/A(t) = \{[\overline{X}^T(t) + \tau \cdot Int(t)]/[Int(t) + \frac{dp(t)}{(1 - \tau)}]\}/A(t)$

Table 5.6

Matrix of Simple Correlation Coefficients

	V/A	$\overline{X}^T$/A	$\Delta\overline{X}^T$/A	$\overline{I}$/A	FCC/A
			1963		
V/A	1.000	.714	.208	.0294	.262
$\overline{X}^T$/A		1.000	.336	.071	.294
$\Delta\overline{X}^T$/A			1.000	.833	.042
$\overline{I}$/A				1.000	.029
FCC/A					1.000
			1965		
V/A	1.000	.717	.311	.046	.255
$\overline{X}^T$/A		1.000	.447	.091	.413
$\Delta\overline{X}^T$/A			1.000	.875	.052
$\overline{I}$/A				1.000	.005
FCC/A					1.000
			1967		
V/A	1.000	.740	.685	.192	.096
$\overline{X}^T$/A		1.000	.594	.003	.428
$\Delta\overline{X}^T$/A			1.000	.655	.185
$\overline{I}$/A				1.000	-.013
FCC/A					1.000
			1969		
V/A	1.000	.637	.459	.299	.170
$\overline{X}^T$/A		1.000	.560	.245	.360
$\Delta\overline{X}^T$/A			1.000	.792	-.040
$\overline{I}$/A				1.000	-.204
FCC/A					1.000
			1971		
V/A	1.000	.664	.368	-.017	.236
$\overline{X}^T$/A		1.000	.543	-.025	.405
$\Delta\overline{X}^T$/A			1.000	.644	-.090
$\overline{I}$/A				1.000	-.385
FCC/A					1.000
			1973		
V/A	1.000	.506	.109	-.120	.231
$\overline{X}^T$/A		1.000	.519	.173	.285
$\Delta\overline{X}^T$/A			1.000	.303	-.236
$\overline{I}$/A				1.000	-.424
FCC/A					1.000

Results of Regressions

Estimated coefficients of equation (5.11) are reported in Table 5.7. The estimated inverse of the average capitalization rate b_1 has the correct sign and is highly significant in every year. Furthermore, the reciprocal of b_1 reported in column (2) of the tables follows quite well the long-term U.S. Treasury and Aa Utility bond rates listed in columns (3) and (4). The estimated growth coefficient, b_2, is statistically significant in only one year, 1967, and its sign is incorrect.

There are several possible explanations for the poor results associated with b_2. They are: (1) each year's coefficients may truly reflect investors' consensus of the duration of growth for electric utilities; (2) the theoretical specification of growth in the valuation model is wrong; (3) the numerical estimates for expected growth in assets and expected incremental earnings do not represent investors' consensus estimates; or (4) multicollinearity between the estimates of investment in assets and expected incremental earnings (see Table 5.6) causes unstable coefficients. More likely than not, multicollinearity plays a role in the imprecise growth coefficients. However, it is not surprising that b_2 lacks significance. Utilities are generally not considered "growth" companies. This, of course, flies in the face of the empirical results of Litzenberger and Rao [32] and Gordon [26]. However, these studies were concerned with the relationship between growth in earnings per share and stock price, not between tax-adjusted earnings and total firm value.

Table 5.7

Nonlinear Regression of Value on Expected Tax-adjusted Earnings, Expected Change in Tax-adjusted Earnings and Expected Investment in Total Assets (N = 69)

$$\frac{V}{A} = \beta_1 \frac{\overline{X}^{\tau}}{A} + \beta_2 \frac{\overline{I}}{A} + \beta_1 \beta_2 \frac{\Delta\overline{X}^{\tau}}{A} + u$$

	(1)	(2)	(3)	(4)	(5)	(6)
Year	b_1	$\frac{Z}{100/b_1}$	Long Term Treasury Yield*	Average Utility Aa Coupon Rate*	b_2	R^2
1963	19.294	5.18	4.08	4.33	.828	.51
	(1.192)				(.473)	
1965	20.612	4.85	4.26	4.54	.311	.51
	(.858)				(.335)	
1967	16.113	6.21	4.94	5.86	.784	.61
	(.610)				(.316)	
1969	14.253	7.02	6.28	7.56	.675	.42
	(1.105)				(.548)	
1971	12.758	7.84	6.12	7.67	.588	.47
	(1.126)				(.477)	
1973	10.799	9.26	7.00	7.74	.635	.28
	(.938)				(.362)	

Note: Numbers in parentheses are standard errors.

*Solomon Brothers Yield Book.

Approximate fit. The square of the multiple correlation coefficient, R^2, is reported in column (6) of Table 5.7. For nonlinear regression R^2 approximately measures the proportionate reduction of total variation in the dependent variable that is associated with the use of the set of explanatory variables. The word approximately is necessary for two reasons. First, it must be assumed that a global minimum (convergence) has been obtained. It is demonstrated below that this requirement appears satisfied. The second reason is more limiting. Equation (4.46) indicates that after iteration j unexplained variation is the sum of the squared error term u_{ij}

$$SS_j(\underline{b}) = \sum_{i=1}^{n} \Big[V_i - f^j(\underline{G};\, b_{1j},\, b_{2j}) - (G_{1i} + b_{2j}G_{3i})\, A_{1j} - (G_{2i} - b_{1j}G_{3i})\, A_{2j} \Big]^2 \qquad (5.12)$$

The estimate obtained for $SS_j(\underline{b})$, the unexplained variation, is valid only to the extent that at convergence its linarized form (equation (5.12)) provides a good approximation to the true model (see equation (5.1)). There is no assurance that this requirement is satisfied. However, the nonlinearity in (5.1) is not of very high order--b_1b_2--and thus it seems reasonable to conclude that at convergence approximation to the true model is adequate.

Of course, $SS_j(\underline{b})$ must also be at a global minimum on iteration j. To be reasonably assured of this, extreme initial values of each coefficient were used ranging from 0.005 to 5000.

The regressions for each of the years were replicated four times, one for each combination of initial values. For every regression the estimated coefficients were identical to those reported in Table 5.7. The maximum number of iterations required was 7 compared to 3 or 4 when more reasonable initial values were used.

Inclusion of a Constant

To substantiate the belief that equation (5.11) should not include an intercept, the regressions were repeated without suppressing the constant term. The results, reported in Table 5.8, neither support nor reject including a constant term in the statistical equation. However, the fact that b_o is significantly positive in 3 of the 6 years may result from measurement errors in $\overline{X}^{\tau}$. To fit equation (5.11) when errors exist in the measurement of expected earnings, will result in an earnings coefficient which is less than its true value, and more so, the larger the variance of the error of measurement. Also, Miller and Modigliani correctly pointed out that this downward asymptotic bias in the earnings coefficient, ". . . will be greater if the earnings coefficient was calculated with a constant in the regression equation because by flattening the slope of the regression line, the bias tends to produce a positive intercept even if none really exists."[4]

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[4] See Miller and Modigliani [39, p. 352].

Table 5.8

Nonlinear Regression of Value on Expected Tax-adjusted Earnings, Expected Change in Tax-adjusted Earnings, Expected Investment in Total Assets and a Constant (N = 69)

$$\frac{V}{A} = \beta_o + \beta_1 \frac{\overline{X}^{\tau}}{A} + \beta_2 \frac{\overline{I}}{A} + \beta_1\beta_2 \frac{\Delta\overline{X}^{\tau}}{A} + u$$

	(1)	(2)	(3)	(4)	(5)
Year	b_o	b_1	$\frac{Z}{100/b_1}$	b_2	R^2
1963	-.208	25.162	3.97	.085	.50
	(.190)	(3.098)		(.247)	
1965	-.091	23.188	4.31	-.407	.51
	(.179)	(2.902)		(.266)	
1967	.072	14.601	6.85	1.036	.61
	(.104)	(1.651)		(.341)	
1969	.243	10.955	9.13	.528	.42
	(.110)	(1.898)		(.356)	
1971	.352	9.088	11.00	-.004	.46
	(.080)	(1.217)		(.264)	
1973	.520	5.547	18.03	-.351	.29
	(.070)	(1.088)		(.169)	

Note: Numbers in parentheses are standard errors.

Instrumental Variables Method

One approach to the errors in variables problem that provides consistent estimators is the instrumental variables method. Implementation of this method requires that a new variable be found that is uncorrelated with both the measurement errors of $\overline{X}^{\tau}$ and u_i, the residual error term of (5.11). Furthermore, the instrumental variable must be positively correlated with tax-adjusted earnings when the latter is measured without error. The selection of specific instrumental variables is highly subjective and for this reason, the variables introduced by M & M in [39] were selected for use here.

Estimation. Fitting equation (5.11) with instrumental variables requires two steps (stages). In the first stage estimated total tax-adjusted earnings are regressed on the selected instrumental variables,

$$\frac{\overline{X}_i^{\tau}}{A_i} = \delta_o + \delta_1 \frac{1}{A_i} + \delta_2 \frac{\Delta A_i}{A_i} + \delta_3 \frac{D_i}{A_i} + \delta_4 \frac{Q_i}{A_i} + \delta_5 \frac{Div_i}{A_i} + v_i \qquad (5.13)$$

where

$1/A_i$ = a size variable

ΔA_i = 4 year average growth

D_i = total debt at market value

Q_i = total preferred stock at market value

Div_i = total dividend payment

v_i = a random disturbance term

In the second stage of the regression fitted values of tax-adjusted earnings calculated as the expected value of equation (5.13) are used in equation (5.11).

Results. The results of the second stage regressions are presented in Tables 5.9 and 5.10. Table 5.9 reports the estimated coefficients of equation (5.11) with the constant term unconstrained. As can be observed in column (1), b_o is not significantly different from zero in any year. Thus, the contention that no constant term should appear in the statistical equation is supported. Table 5.10, which contains the results of the regression when the constant term is constrained to zero, reports a b_1 coefficient that is highly significant. However, it is disappointing to note that the estimate of Z is consistently below the Aa coupon rate.

The growth coefficient b_2 has the correct sign in every year and is highly significant in 1963, 1965, 1971, and 1973. It is significantly less than zero at the 5 percent level in 1969. An implied estimate of the number of years of growth opportunities, calculated as

$$\overline{T} = -b_2 \ (1 + Z) \tag{5.14}$$

is reported for each year in column (6) of Table 5.10. All estimates of T are less than one year indicating that investors in electric utilities do not capitalize growth opportunities beyond the current year.

Table 5.9

Nonlinear Second Stage Regression of Value on Instrumental Variables, Expected Change in Tax-adjusted Earnings, and Expected Investment in Total Assets With A Constant Term (N = 69)

$$\frac{V}{A} = \beta_o + \beta_1 \frac{X\tau}{A} + \beta_2 \frac{I}{A} + \beta_1\beta_2 \frac{\Delta X^{\tau}}{A} + u$$

Year	(1) b_o	(2) b_1	(3) $\frac{Z}{100/b_1}$	(4) b_2	(5)* R^2
1963	-.244 (.277)	27.889 (4.418)	3.59	-.614 (.201)	.40
1965	-.112 (.252)	24.982 (3.844)	4.00	-.611 (.216)	.40
1967	-.044 (.201	18.770 (2.248)	5.33	-.238 (.236)	.38
1969	-.034 (.159)	16.500 (2.502)	6.06	-.274 (.154)	.35
1971	.105 (.177)	16.634 (2.533)	6.01	-.239 (.112)	.47
1973	-.101 (.137)	13.405 (2.022)	7.46	-.214 (.066)	.41

Note: Numbers in parentheses are standard errors.

* Pertains to second stage only.

Table 5.10

Nonlinear Second Stage Regression of Value on Instrumental Variables, Expected Change in Tax-adjusted Earnings, and Expected Investment in Total Assets With Constant Term Constrained to Zero
(N = 69)

$$\frac{V}{A} = \beta_1 \frac{\overline{X}^{\tau}}{A} + \beta_2 \frac{\overline{I}}{A} + \beta_1\beta_2 \frac{\Delta\overline{X}^{\tau}}{A} + u$$

Year	(1) b_1	(2) $\frac{Z}{100/b_1}$	(3) Long Term Treasury Yield*	(4) Average Utility Aa Coupon Rate*	(5) b_2	(6) $\overline{T}$	(7) R^2†
1963	23.961 (.564)	4.17	4.08	4.33	-.685 (.193)	.71	.40
1965	23.251 (.623)	4.30	4.26	4.54	-.638 (.204)	.66	.40
1967	18.107 (.505)	5.52	4.94	5.86	-.252 (.215)	.27	.39
1969	15.956 (.491)	6.27	6.28	7.56	-.274 (.156)	.29	.37
1971	15.119 (.324)	6.61	6.12	7.67	-.277 (.094)	.30	.48
1973	13.256 (.223)	7.54	7.00	7.74	-.216 (.062)	.23	.42

Note: Numbers in parentheses are standard errors.

* Solomon Brothers Yield Book.

† Pertains to second stage only.

Evaluation of the instrumental variable approach. The major advantage of the instrumental variable approach is that consistent estimators of the coefficients of equation (5.11) have been estimated. Typically this consistency occurs at the expense of less precise estimators. However, in this case the instrumental variable approach has provided more precise estimates of β_1 and β_2, i.e., the standard errors for each coefficient are smaller in every year. The increase in precision can largely be explained by examining the degree of correlation among the independent variables in the second stage listed in Table 5.11. Comparing Table 5.11 with Table 5.6, it is clear that the degree of multicollinearity among the independent variables is considerably reduced, especially between $\Delta\overline{X}^{\tau}/A$ and $\overline{I}/A$. Statistically then, the instrumental variable approach seems to provide superior results. Unfortunately, the values obtained for Z are so low that they cannot be seriously considered as good approximations to the cost of capital. These results will be further explored in the Chapter VI.

The Effect of Financial Leverage on Firm Value

To provide some indication that equation (4.42) correctly specifies total firm value, the relationship between financial leverage and firm value is now examined. The purpose is not to thoroughly investigate the characteristics of financial leverage and firm value. Rather, by adding an additional independent variable to equation (5.11), the correct specification of (4.42), as a model of firm value, is being tested. In the direct cost of

Table 5.11

Matrix of Simple Correlation Coefficients of Second Stage Independent Variables

	V/A	$\overline{X}'$/A	$\Delta\overline{X}'$/A	$\overline{I}$/A	FCC/A
1963					
V/A	1.000	.553	-.402	.064	.262
$\overline{X}'$/A		1.000	.024	-.119	.372
$\Delta\overline{X}'$/A			1.000	.059	-.001
$\overline{I}$/A				1.000	.098
FCC/A					1.000
1965					
V/A	1.000	.572	-.287	-.028	.255
$\overline{X}'$/A		1.000	.170	-.243	.465
$\Delta\overline{X}'$/A			1.000	-.032	-.025
$\overline{I}$/A				1.000	-.046
FCC/A					1.000
1967					
V/A	1.000	.617	-.379	.195	.098
$\overline{X}'$/A		1.000	-.182	-.101	.415
$\Delta\overline{X}'$/A			1.000	.127	-.223
$\overline{I}$/A				1.000	-.128
FCC/A					1.000
1969					
V/A	1.000	.590	-.114	.277	[illegible]
$\overline{X}'$/A		1.000	.219	.215	.452
$\Delta\overline{X}'$/A			1.000	.133	-.024
$\overline{I}$/A				1.000	-.201
FCC/A					1.000
1971					
V/A	1.000	.650	-.472	-.026	.238
$\overline{X}'$/A		1.000	-.323	-.062	.443
$\Delta\overline{X}'$/A			1.000	.213	-.335
$\overline{I}$/A				1.000	-.406
FCC/A					1.000
1973					
V/A	1.000	.369	[illegible]	-.113	.231
$\overline{X}'$/A		1.000	-.172	.219	[illegible]
$\Delta\overline{X}'$/A			1.000	.360	.235
$\overline{I}$/A				1.000	-.429
FCC/A					1.000

capital model any benefit resulting from financial leverage, not passed through to consumers, is reflected in tax-adjusted earnings and changes in those earnings. Thus, if equation (5.11) correctly specifies total firm value, the expected value of the coefficient of an additional variable (degree of financial leverage) will be zero.

Measuring Financial Leverage

The degree of financial leverage is usually measured as either a stock or flow variable. The former includes various ratios of debt relative to total value, total assets, or equity. Flow variables are ratios such as times interest earned or fixed charge coverage. Stock ratios, while indicating the proportion of debt in a capital structure, indicate little about the debt capacity of a firm. Furthermore, including debt on both sides of the regression equation (total value of course contains debt), and also including total value on both sides (in the denominator of the debt ratio) raises the problem of spurious correlation between dependent and independent variables. To avoid such problems the degree of financial leverage is here indicated as the fixed charge coverage

$$FCC = \frac{\text{Earnings Before Interest and Taxes}}{\text{Total Interest Payments} + \dfrac{\text{Preferred Stock Dividends}}{(1 - \tau)}} \tag{5.15}$$

Adding FCC/A to equation (5.11) provides the altered regression equation

$$\frac{V}{A} = \beta_1 \frac{\bar{X}^{\tau}}{A} + \beta_2 \frac{\bar{I}}{A} + \beta_1\beta_2 \frac{\overline{\Delta X}^{\tau}}{A} + \beta_3 \frac{FCC}{A} + u \qquad (5.16)$$

Regression Results, Entire Sample

Table 5.12 contains the results of fitting equation (5.16) by the nonlinear instrumental variable method presented above. At the 5 percent level of significance b_3 is different than zero only in 1967, tentatively confirming the null hypothesis that financial leverage has no impact on the total market value of electric utilities.

The Equal Business Risk Assumption

Of the cost of capital studies reviewed in Chapter III, only McDonald [36] attempted to ascertain the homogeneity of his sample. The other studies merely assumed, as was done in the previous section, that the equal business risk assumption was adequately satisfied by industry categorization. It was also pointed out in Chapter III that such classification does not assure groups of firms equivalent in business risk. Thus, prerequisite to concluding that the model is correctly specified, is a presentation of evidence demonstrating that the sample is homogeneous in business risk.

Appendix B examines the homogeneity of the firms in the sample by testing the hypothesis

$$\Phi_i(R) = \Phi_j(R) \qquad (5.17)$$

where

Table 5.12

Nonlinear Second Stage Regression of Value on Instrumental Variables, Expected Change in Tax-adjusted Earnings, Expected Investment in Total Assets and Fixed Charge Coverage (N = 69)

$$V = \beta_1 \frac{\overline{X}^\tau}{A} + \beta_2 \frac{\overline{I}}{A} + \beta_1 \beta_2 \frac{\Delta \overline{X}^\tau}{A} + \beta_3 \frac{FCC}{A} + u$$

	(1)	(2)	(3)	(4)
Year	b_1	b_2	b_3	R^2†
1963	23.630	-.691	1.321	.40
	(.701)	(.196)	(1.682)	
1965	23.486	-.646	-.890	.40
	(.801)	(.204)	(1.961)	
1967	18.955	-.358	-3.301	.43
	(.635)	(.206)	(1.648)	
1969	16.452	-.324	-2.531	.40
	(.568)	(.152)	(1.591)	
1971	15.628	-.349	-3.154	.48
	(.453)	(.100)	(2.042)	
1973	13.408	-.238	-1.286	.42
	(.308)	(.069)	(1.793)	

Note: Numbers in parentheses are standard errors.

† Pertains to second stage only.

R = annual rate of return before interest and taxes on total assets

$\Phi_i(R)$ = the distribution of returns for firm i.

Upon rejection of (5.17), an ad hoc screening procedure is devised to identify those firms that do satisfy condition (5.17). This results in the formation of two subsamples, each with 14 firms and labeled L1 and H2 (see Appendix C).

Regression Results on the Subsample

The results of fitting equation (5.16) on each of the equal risk groups, L1 and H2, are reported in Tables 5.13 and 5.14 respectively. The magnitude of b_3 relative to its standard error indicates that b_3 is not significantly different from zero. Thus the minor hypothesis cannot be rejected.

Summary

This chapter has demonstrated that cost of capital numbers can be directly estimated from the valuation model developed in Chapter IV. The instrumental variable nonlinear regressions of the hypothesized valuation model have provided consistent estimates for each of the sample years. The numbers obtained for b_1 in Table 5.10 are 3 to 6 times larger than their respective standard errors. Furthermore, the cost of capital estimates track quite well the prevailing interest rates. Restriction of the constant term to zero has been empirically justified. The use of instrumental variables reduced the degree of multicollinearity to acceptable limits. Also, when the instrumental variable method was employed, b_2, the

Table 5.13

Group L1

Nonlinear Second Stage Regression of Value on Instrumental Variables, Expected Change in Tax-adjusted Earnings, Expected Investment in Total Assets and Fixed Charge Coverage (N = 14)

$$V = \beta_1 \frac{\overline{X^\tau}}{A} + \beta_2 \frac{\overline{I}}{A} + \beta_1\beta_2 \frac{\Delta\overline{X^\tau}}{A} + \beta_3 \frac{FCC}{A} + u$$

Year	(1) b_1	(2) b_2	(3) b_3	(4) R^2†
1963	22.576 (1.580)	-.278 (.393)	-.672 (5.022)	.29
1965	21.049 (1.547)	-.056 (.452)	.595 (5.561)	.29
1967	17.579 (1.098)	-.054 (.332)	-3.172 (4.251)	.50
1969	14.032 (.838)	.360 (.268)	-3.277 (3.167)	.56
1971	14.163 (.703)	-.061 (.197)	-2.734 (3.579)	.56
1973	11.848 (.707)	-.034 (.194)	4.569 (5.356)	.37

Note: Numbers in parentheses are standard errors.

† Pertains to second stage only.

Table 5.14

Group H2

Nonlinear Second Stage Regression of Value on Instrumental Variables, Expected Change in Tax-adjusted Earnings, Expected Investment in Total Assets and Fixed Charge Coverage (N = 14)

$$V = \beta_1 \frac{\overline{X}^{\tau}}{A} + \beta_2 \frac{\overline{I}}{A} + \beta_1\beta_2 \frac{\Delta\overline{X}^{\tau}}{A} + \beta_3 \frac{FCC}{A} + u$$

Year	(1) b_1	(2) b_2	(3) b_3	(4) R^2†
1963	20.715 (1.506)	.223 (.772)	4.237 (4.304)	.25
1965	19.963 (1.334)	.474 (.389)	2.846 (4.340)	.33
1967	16.858 (1.122)	.611 (.506)	.162 (4.158)	.39
1969	16.557 (2.244)	-.182 (.599)	-3.356 (4.939)	.22
1971	14.696 (1.145)	-.125 (.227)	-.400 (4.987)	.33
1973	10.635 (1.131)	.414 (.248)	7.640 (7.756)	.46

Note: Numbers in parentheses are standard errors.

† Pertains to second stage only.

growth coefficient always had the hypothesized sign and was significant in all but one of the regressions.

To test the impact of financial leverage on total firm value, it was necessary to examine rate of return distributions for firms in the sample (see Appendix B). The minor hypothesis of homogeneous business risk among electric utilities was rejected. However, two subsamples were formed based on equal rate of return distributions. A fixed charge coverage ratio was proposed as a measure of financial leverage for the firm. When this ratio was included in the statistical equation as an independent variable, it was found to be not significantly different than zero. Thus the minor hypothesis that financial leverage has no impact on firm value cannot be rejected.

CHAPTER VI

CONCLUSION

Rather than presenting an extensive summary of a study already perhaps excessive in length, this chapter sets forth only what appear to be principal points of the thesis.

The Average Cost of Capital

An equation representing total firm value under market conditions of equilibrium has been proposed in the proceeding pages (see Chapter IV). The model is traditional in its structure in that value is determined by capitalizing the earnings of the firm by the average cost of capital.

Features of the Model

Several characteristics of the model distinguish it from other valuation models.

Tax-adjusted earnings. By capitalizing income accruing to all investors in the firm, the impact of financial leverage is reflected in the variable $\overline{X^{\tau}}$. Earlier models have generally examined the income expected by common stockholders in isolation, or, as a separate component from preferred stock dividends, interest payments, and tax benefits arising therefrom.

Growth. Expected growth in tax-adjusted earnings is included in the valuation model. Furthermore, any increase in the tax subsidy resulting from growth is, contrary to other growth earnings. The growth component of firm value, while similar in structure to earlier models, reflects the nonlinearity of the term, both in the theoretical development and the empirical testing.

Regulation. The valuation model was demonstrated to be compatible with the process of utility rate of return regulation. Specifically, assuming that the regulatory objective is to maintain existing firm value, the model provides a tool to establish allowed rates of return.

Estimating the Average Cost of Capital

The average cost of capital was estimated using nonlinear least squares regression (see Tables 5.7 and 5.8) and by instrumental variable nonlinear least squares regression (see Tables 5.9 and 5.10). The instrumental variable approach was used because of what was believed to be measurement errors in $\overline{X}^{\tau}$, causing signficantly positive intercepts in the latter three years of the study. An equally plausible explanation though is that the statistical model without a constant term is, in some years, misspecified, e.g., perhaps the size of a firm has an impact upon its total value.

Unfortunately, when the instrumental variable approach was employed, the estimates of Z were too low to be considered as approximations of the costs of capital. To explain the high values for b_1, we must consider the quality of the instruments used to estimate total tax-adjusted earnings. Whether or not better instruments can be found is largely an empirical question. However, the use of dividends as an instrument to explain total earnings would seem, a priori, to be a bad choice because of the simultaneity which exists between dividends and total earnings, i.e., which variable is the predictor? Thus if the instrumental variable approach is to be used with this model, improved instruments are required.

Average cost of capital, Z. Rejecting the use of instrumental variables, leaves no alternative but to accept as better results the numbers reported in Table 5.7. These numbers are also questionable. The positive constant term for three years indicates a misspecification of the statistical equation. Thus, it must be concluded that the proposed valuation model is, at present, less than satisfactory for explaining total firm value. However, it is clear that the disappointing performance of the model remains an empirical problem which can be solved.

Additional Results

Several additional results are worth noting.

Estimate of growth. When the instrumental variable approach was used, the estimated growth coefficient had the hypothesized negative sign in every year and was significant in 5 of the 6 years. However, the number of years of tax-adjusted earnings growth implied by the coefficient was quite low, indicating that growth contributes little to the total value of electric utilities, at least in the sample years.

Financial leverage. A test for the impact of financial leverage was conducted by adding Fixed Charge Coverage to the regression equation as an additional independent variable. As hypothesized, the estimated coefficient of Fixed Charge Coverage was not statistically significant in any year. Thus, it was concluded that the total value model was correctly specified with respect to financial leverage for the sample.

Homogeneous business risk. As a designator of business risk, industrial classification was shown to be invalid when business risk was identified by the distribution of the ratio of earnings before interest and taxes to total assets. However, 2 groups of firms were identified such that each group was characterized by identical rate of return distributions. Unfortunately, the degrees of correlation across firm's rates of return were desparate to the extent that the formation of true equal business risk subsamples could not be considered successful.

Extensions

There are several areas to which extension of the current analysis points.

Estimation of Independent Variables

A logical extension of the effort reported here is an attempt to improve the set of ex ante estimates of the independent variables. Hopefully improved estimates of $\overline{X}^{\tau}$, $\overline{I}$, and $\Delta\overline{X}^{\tau}$ will provide better estimates of Z.

Financial Leverage

A second and perhaps more important area for future research is the use of the model to test what impact financial leverage has on total firm value. The test in Chapter V, while providing casual evidence about the correct specification of the statistical model, leaves many questions unanswered. For example, would the same results obtain for a sample of industrial firms? Can a larger sample of firms homogeneous in business risk be selected?

Full Equilibrium

The most important possible extension of the model of course is a full equilbrium framework resting not on the restrictive assumptions of the flawed Capital Asset Pricing Model, but rather on the simple concept of arbitrage in a perfectly competitive market.

Summary

The last two decades have demonstrated an increased dependence of our society on electrical energy. It has also been a time of generally rising capital costs which, if energy is to be priced efficiently, must be accurately reflected in the rate of return that electric utilities are permitted to earn. The purpose of modeling the valuation of an electric utility has been to provide a better estimate of the cost of capital to both managers and regulators of electric utilities. The obvious usefulness of the model is determined by how well it predicts the average cost of capital, not with how restrictive its underlying assumptions seem to be. Given the low estimates obtained in Chapter V, it must be concluded that the model does not predict very well. Thus additional verification of the total valuation model's correct specification (including comparing its estimates of Z to other model's estimates) is required if the model is ever to be used in the regulatory process for electric utilities.

APPENDIX A

ELECTRIC UTILITIES INCLUDED IN THE SAMPLE

Firm Number	Firm Name
1	Arizona Public Service Co.
2	Baltimore Gas and Electric
3	Central Hudson Gas and Elec.
4	Central Maine Power Co.
5	Cincinnati Gas and Electric
6	Cleveland Electric Illum.
7	Columbus and Southern Ohio
8	Consolidated Edison of NY
9	Dayton Power and Light
10	Delmarva Power and Light
11	Duquesne Light Co.
12	Kansas City Power and Light
13	Long Island Lighting
14	Nevada Power Co.
15	New York State Elec. and Gas
16	Niagara Mohawk Power
17	Orange and Rockland Utilities
18	Pacific Gas and Electric
19	Pacific Power and Light
20	Pennsylvania Power and Light
21	Potomac Electric Power
22	Public Service Co. of Colo.
23	Public Service Co. of NH
24	Public Service Elec. and Gas
25	Puget Sound Power and Light
26	Rochester Gas and Electric
27	San Diego Gas and Electric
28	Southern Calif. Edison Co.
29	Toledo Edison Company
30	Tucson Gas and Electric
31	Union Electric Co.
32	United Illuminating Co.
33	Utah Power and Light
34	Virginia Electric and Power
35	Washington Water Power

Firm Number	Firm Name
36	Atlantic City Electric
37	Boston Edison Co.
38	Carolina Power and Light
39	Central Illinois Light
40	Central Ill. Public Service
41	Central Louisiana Electric
42	Commonwealth Edison
43	Consumers Power Co.
44	Detroit Edison Co.
45	Duke Power Co.
46	El Paso Electric Co.
47	Florida Power and Light
48	Florida Power Corp.
49	Gulf States Utilities Co.
50	Hawaiian Electric Co.
51	Houston Lighting and Power
52	Idaho Power Co.
53	Illinois Power Co.
54	Indianapolis Power and Light
55	Interstate Power Co.
56	Iowa-Illinois Gas and Elec.
57	Iowa Power and Light
58	Iowa Public Service Co.
59	Kansas Gas and Electric
60	Kentucky Utilities Co.
61	Louisville Gas and Electric
62	Montana-Dakota Utilities
63	Montana Power Co.
64	Northern Indiana Public Service
65	Northern States Power
66	Oklahoma Gas and Electric
67	Philadelphis Electric Co.
68	Public Service Co. of Ind.
69	Public Service Co. of NM

APPENDIX B

A TEST FOR EQUAL BUSINESS RISK

A test for the impact of financial leverage on firm value requires a sample homogeneous in its degree of business risk. This appendix proposes a test for identifying firms equal in business risk.

Equal Business Risk: Entire Sample

Defining Equal Business Risk

Let λ_i be a proportionality factor at the beginning of period 1 such that

$$A_i(1) = \lambda_i A_j(1) \qquad (B.1)$$

where A_i and A_j are the total undepreciated assets of firms i and j respectively. The second Fama-Miller equal risk condition (see equation (2.67b)) requires that investment in both firms subsequent to period 1 be equated by λ_i so that for all future periods

$$A_i(t) = \lambda_i A_j(t) \qquad (B.2)$$

The first Fama-Miller equal risk condition (see equation (2.67a)) stipulates that the income of the two firms is also equated by the same proportionality factor such that

$$X_i(t) = \lambda_i X_j(t) \tag{B.3}$$

For both conditions to hold simultaneously, then

$$R_i(t) = R_j(t) = \frac{X_i(t)}{A_i(t)} = \frac{\lambda_i X_j(t)}{\lambda_i A_j(t)} \tag{B.4}$$

where $R_i(t)$ and $R_j(t)$ are the annual rates of return before interest and taxes for firms i and j respectively.

Equation (B.4) is an equal risk class condition equivalent to the Fama-Miller conditions (see (2.67a) and (2.67b)). Unfortunately, it is confining to the extent that N firms will always be grouped into no fewer than N risk classes. However, given that R_i is stochastic, a less confining interpretation of (B.4) requires only that

$$\Phi_i(R) = \Phi_j(R) \tag{B.5}$$

where $\Phi_i(R)$ is the distribution of returns for firm i.

Testing for Equal Business Risk

Simple histograms of the returns of several firms such as those in Figure B.1 indicate that realized rates of return for many of the firms in the sample may be normally distributed.

Tests for normal distribution. Two tests for normality were utilized: A Kolmogorov-Smirnov test and a moments test. The Kolmogorov-Smirnov test, the more conservative of the two, compares a sample cumulative distribution $S_i(R)$ with the theoretical normal cumulative distribution $F_i(R)$ over N observations. The sample test statistic is calculated as

1. NIAGARA MOHAWK POWER (1955-1974)

```
5                         *
4                         *       *
3                     *   *       *   *
2                 *   *   *       *   *
1         *   *   *   *   *       *   *         *

          ----------------------------------------

NUMBER    1   1   2   3   5   0   4   3   0     1

Cell      1   2   3   4   5   6   7   8   9    10
Range:    5.3 to 8.3, Interval size:  0.3
Mean =    6.8, Variance = 0.5
```

2. UNITED ILLUMINATING (1955-1974)

```
6                             *
5                             *
4                     *       *
3                     *   *   *       *
2         *           *   *   *       *
1         *       *   *   *   *       *         *

          ----------------------------------------

NUMBER    2   0   1   4   3   6   0   3   0     1

Cell      1   2   3   4   5   6   7   8   9    10
Range:    6.1 to 13.1, Interval size:  0.7
Mean =    9.5, Variance = 2.8
```

3. TOLEDO EDISON COMPANY (1955-1974)

```
4                         *
3                     *   *   *   *       *
2                     *   *   *   *       *
1         *       *   *   *   *   *   *   *     *

          ----------------------------------------

NUMBER    1   0   1   3   4   3   3   1   3     1

Cell      1   2   3   4   5   6   7   8   9    10
Range:    7.0 to 11.2, Interval size:  0.4
Mean =    9.3, Variance = 1.0
```

Figure B.1

Rate of Return Histograms (N = 20)

$$D = \max_{\text{all } i} \left| F_i(R) - S_i(R) \right| \qquad (B.6)$$

with N degrees of freedom equal to either the number of observations or number of cells, whichever is less. The moments test, developed by Pearson and Hartley [48], is based on the third and fourth central moments of a distribution. It requires that test statistics $\tilde{\nu}_1$ and $\tilde{\nu}_2$ be calculated and compared to critical values.[5] For a normal distribution the null hypothesis is

$$\tilde{\nu}_1 = \frac{\delta_3}{\delta_2^{3/2}} = 0 \qquad (B.7)$$

and

$$\tilde{\nu}_2 = \frac{\delta_4 - 3\delta_2}{\delta_2^2} = 0 \qquad (B.8)$$

where

$$\delta = \int R \cdot f(R)dr$$

$$\delta_2 = \int (R - \delta)^2 f(R)dR$$

$$\delta_3 = \int (R - \delta)^3 f(R)dR$$

$$\delta_4 = \int (R - \delta)^4 f(R)dr$$

Results of the tests. Using the Kolmogorov-Smirnov test at the 5 percent level of significance, it was not possible to reject the hypothesis of normally distributed rates of return for any of the firms in the sample. However, histograms of realized rates of return for several of the firms (see Figure B.2) are skewed to the left. Also, the rate of return histogram for Indianapolis Power and Light, in addition to being negatively skewed, is disturbingly flat. The appearance of these and other histograms prompted the use of the moments test to evaluate the significance of the third and fourth moments of the distributions. Table B.1 provides a summary of means, variances, test statistics for skewness, test statistics for kurtosis, and the Kolmogorov-Smirnov D statistics for all firms in the sample. While all D statistics are less than the critical value of 0.294, the distribution of eleven firms are significantly skewed, two have significant kurtosis, and four firms have distributions in which both of the higher moments are significant. Since the distributions of rates of return of a given risk class must be identical, the seventeen firms with higher moments in their rate of return distributions can now be eliminated prior to further testing. Also, in view of the disparate rate of return distributions, the hypothesis that electric utility firms are equal in business risk is rejected.

---

[5] See Pearson and Hartley [48], and Jones [28], who developed critical values for small sample sizes.

Identifying Equivalent Normal Distributions

Two normal distributions are identical if the mean and variance of the distributions are equal.

1. PENNSYLVANIA POWER & LIGHT (1955-1974)

```
6                                             *
5                                             *
4                                     *       *
3                             *       *   *   *
2                             *       *   *   *
1         *       *   *       *   *   *   *   *

          --------------------------------------

NUMBER    1   0   1   1   0   3   1   4   3   6

Cell      1   2   3   4   5   6   7   8   9   10
Range:    6.0 to 10.0, Interval size:  0.4
Mean -    8.8, Variance = 1.1
```

2. MONTANA-DAKOTA UTILITIES (1955-1974)

```
6                                         *
5                                     *   *
4                                     *   *   *
3                                     *   *   *
2                         *       *   *   *   *
1         *               *       *   *   *   *

          --------------------------------------

NUMBER    1   0   0   0   2   0   2   5   6   4

Cell      1   2   3   4   5   6   7   8   9   10
Range:    9.0 to 13.1, Interval size:  0.4
Mean =    12.0, Variance = 0.86
```

3. INDIANAPOLIS POWER & LIGHT (1955-1974)

```
4                         *       *   *
3                         *       *   *   *
2                         *   *   *   *   *   *
1         *               *   *   *   *   *   *

          --------------------------------------

NUMBER    1   0   0   0   4   2   4   4   3   2

Cell      1   2   3   4   5   6   7   8   9   10
Range:    8.1 to 12.2, Internal size:  0.4
Mean =    10.7, Variance = 0.81
```

Figure B.2

Rate of Return Histograms

Table B.1

Test Results for Equal Rate of Return Distributions

Firm	Avg. $[\frac{EBIT}{A} \times 100]$	Var. $[\frac{EBIT}{A} \times 100]$	Unbiased Measure of Skewness $(\tilde{\nu}_1)^a$	Unbiased Measure of Kurtosis $(\tilde{\nu}_2)^b$	Kolmogorov-Smirnov Statistic $(D)^c$
1	7.05679	.70259	.41	-.80	.14214
2	9.75424	1.14786	-1.12*	.91	.16268
3	7.73524	.54834	-1.12*	1.05	.11424
4	8.40084	.73940	-.38	-.97	.10841
5	10.68419	1.25584	-1.04*	.98	.12396
6	10.57804	1.74640	-.11	-.94	.08589
7	8.40659	1.55223	-.55	-.60	.14973
8	6.08714	.50616	.33	-.95	.15984
9	10.04369	3.33750	-.54	-.93	.12661
10	8.93359	.87272	-.50	-.46	.13274
11	9.67459	.86979	-.74	-.61	.17282
12	9.22144	.53206	-.05	-.50	.09822
13	8.15944	.24205	-.35	-.26	.09009
14	8.54359	1.03947	.85	.62	.19980
15	8.15859	.64841	-.54	-.75	.10545
16	6.78719	.54684	.07	-.57	.11682
17	7.16084	.69865	-.63	.86	.07123
18	8.07954	0.10143	-.20	1.28	.09302
19	6.68759	.36028	.87	.12	.29226
20	8.83754	1.08548	-1.23*	1.11	.16305
21	7.35409	.47133	-.47	-.38	.08245
22	8.20414	.67515	-.07	-.08	.15020
23	7.65514	.27844	-.74	.62	.13161

Table B.1--Continued

Firm	Avg. $[\frac{EBIT}{A}$ x 100]	Var. $[\frac{EBIT}{A}$ x 100]	Unbiased Measure of Skewness ($\tilde{v}_1$)[a]	Unbiased Measure of Kurtosis ($\tilde{v}2$)[b]	Kolmogorov-Smirnov Statistic (D)[c]
24	7.90739	.55681	-1.02	.18	.11536
25	6.56254	.90085	1.19*	.82	.14681
26	7.72834	.38426	-.50	-.58	.12086
27	8.20209	.78022	-.55	-.41	.12691
28	7.84794	.61160	.93	1.64	.13599
29	9.34674	1.00074	-.14	0.0	.08529
30	9.48179	2.34818	.14	.54	.12534
31	7.73049	.44733	-.71	.75	.18719
32	9.48624	2.84900	-.11	.39	.11592
33	7.64184	.16901	-.26	.15	.11870
34	8.87599	1.35160	-1.02	-.15	.19115
35	6.81524	.90640	-.08	-.98	.14976
36	7.86414	.21881	-.24	.32	.10859
37	8.33639	.56016	-.25	-.90	.14263
38	8.87829	.87241	-1.41*	1.80	.13693
39	9.72894	1.04007	-.24	.62	.14520
40	10.99989	1.38608	-1.05*	.29	.18421
41	9.19589	.51477	.63	.40	.10119
42	9.66134	1.23689	-.03	-1.57*	.15471
43	9.03229	1.92699	-1.16*	.95	.11438
44	8.18384	1.29246	.59	-.40	.14013
45	9.38124	2.00057	-1.09*	0.0	.23171
46	11.83529	.60192	-1.17*	2.33	.09313
47	11.55429	2.15657	-.46	-1.01	.13395
48	9.62014	.65132	-1.56*	2.57*	.16226
49	9.47254	.15284	-.55	3.01*	.13653
50	8.35169	.13716	-.10	1.87	.09927
51	12.88744	1.82881	.10	-.12	.17570

Table B.1--Continued

Firm	Avg. $[\frac{EBIT}{A} \times 100]$	Var. $[\frac{EBIT}{A} \times 100]$	Unbiased Measure of Skewness $(\tilde{\nu}_1)^a$	Unbiased Measure of Kurtosis $(\tilde{\nu}_2)^b$	Kolmogorov-Smirnov Statistic $(D)^c$
52	7.76239	.84028	.12	-.82	.07674
53	11.00504	1.90095	-.43	-1.55	.19522
54	10.73504	.81150	-1.10*	2.57*	.06874
55	9.23534	.28773	.43	-.43	.13988
56	10.86789	2.39986	-.57	-.28	.08604
57	8.63189	.26720	-.15	0.0	.12434
58	9.42044	.60418	-.10	-.77	.09779
59	10.35079	.98172	-.97	.84	.09033
60	10.61194	1.74554	-1.06*	.76	.13309
61	11.90769	1.54216	-.42	-.83	.13500
62	8.30054	.41824	.52	-.81	.17449
63	12.04784	.86482	-1.97*	5.04*	.15779
64	10.30094	.57599	-.56	1.17	.07739
65	9.04054	.36380	-.26	-.80	.14430
66	10.38964	.52434	-.17	-1.15	.15189
67	8.73914	.70540	-.40	.35	.12827
68	9.83484	1.65918	-.17	-1.52	.15234
69	10.20099	.41327	-1.53*	3.09*	.21221

Note: An asterisk (*) indicates that the appropriate critical value is greater than the calculated test statistic. Critical values for the test statistics $\tilde{\nu}_1$ and $\tilde{\nu}_2$ are from tables developed by Jones [40]. Critical values for the Kolmogorov-Smirnov test statistic are from Massey [35].

[a] $P\{-1.037 \geq \tilde{\nu}_1 \geq 1.037\} = .95$

[b] $P\{-1.26 \geq \tilde{\nu}_2 \geq 2.49\} = .95$

[c] $P\{D \leq .294\} = .95$

Equal Variance

A procedure for testing if N populations have equal variances is the Bartlett test. Underlying assumptions are that each of the N populations is normally distributed and that the samples from each population are independent and random. The Bartlett test compares the weighted arithmetic average of the sample variances (MSE when weighted by the degrees of freedom) to the weighted geometric average of the same variances (GMSE). That is, if all sample variances are equal, then

$$MSE = \frac{1}{N - 4} \cdot \sum_{j=1}^{r} (N_j - 1)S_j^2 \tag{B.9}$$

equals

$$GMSE = \left[\prod_{j=1}^{r} (S_j^2)^{N_j - 1} \right]^{\frac{1}{N - r}} \tag{B.10}$$

where

N = grand total number of observations equal to $\sum_{j=1}^{r} N_j$

r = number of populations

N_j = number of observations in the jth population

S_j^2 = sample variance of population j.

When population variances are equal, Bartlett has shown that [log(MSE) - log(GMSE)] follows for large sample sizes the χ^2 distribution with (r - 1) degrees of freedom. The test statistic is

$$B = \frac{2.302585}{C} (N - r)(\log_{10}MSE - \log_{10}GMSE) \tag{B.11}$$

where

$$C = 1 + \frac{1}{3(r-1)}\left[\sum_{j=1}^{r} \frac{1}{N_j - 1} - \frac{1}{N - r}\right] \qquad (B.12)$$

and 2.302585 is a factor to convert logarithms from the base ten to the base e.

Test for equal variances. At the one percent level of significance the null hypothesis

$$H_o: \; S_1^2 = S_2^2 = \ldots = S_{52}^2$$

was rejected ($B = 265.64 > \chi^2(.99;\, 51)$). However, by omitting those firms with either extremely large or small variances (a total of four were discarded) and dividing the remaining forty-eight firms approximately in half according to increasing variance, two groups of firms were formed, the variances within each group not being statistically different at the 5 percent level of significance. Tables B.2 and B.3 list firm numbers (corresponding to Appendix A), mean rates of return in ascending order, and the variances of returns for companies in low (L) and high (H) variance groups. Of course the possibility of forming alternate groups exists. The object is to have two groups with many firms in each group.

Equal Means

Analysis of variance and the Scheffé Multiple Comparison Test were used to test the hypothesis that firms within each variance group have equal mean rates of return. For both tests the following assumptions apply:

Table B.2

Rate of Return Means and Standard Deviations of Firms Included in Group L (Low Risk)

Firm Number	Avg. $\left[\frac{EBIT}{A} \times 100\right]$	Var. $\left[\frac{EBIT}{A} \times 100\right]$
8	6.087139	.506160
19	6.687590	.360280
16	6.787189	.546840
17	7.160839	.689650
21	7.354090	.471330
33	7.641840	.169010
23	7.655140	.278440
26	7.728339	.384260
31	7.730490	.447330
28	7.847939	.611600
36	7.864140	.218810
24	7.907390	.556810
15	8.158589	.648410
13	8.159439	.242050
22	8.204140	.675150
62	8.300540	.418240
37	8.336390	.560160
57	8.631889	.267200
65	9.040540	.363800
41	9.195889	.514770
12	9.221439	.532060
55	9.235339	.287730
58	9.420440	.604180
64	10.300940	.575990
66	10.389640	.524340

Grand Mean = 8.2018

Mean Square Error = .4567

Bartlett Coefficient = 28.374

$\chi^2(.95; 24) = 36.42$

Table B.3

Rate of Return Means and Standard Deviations of Firms Included in Group H (High Risk)

Firm Number	Avg. $\left[\frac{EBIT}{A} \times 100\right]$	Var. $\left[\frac{EBIT}{A} \times 100\right]$
35	6.815240	.906400
1	7.056789	.702590
52	7.762389	.840280
44	8.183840	1.292459
27	8.202085	.780220
4	8.400840	.739400
7	8.406590	1.552230
14	8.543590	1.039470
67	8.739140	.705400
34	8.875990	1.351600
10	8.933590	.872780
29	9.346740	1.007400
30	9.481790	2.348180
11	9.674590	.869790
39	9.728939	1.040070
68	9.834840	1.659180
59	10.350789	.981720
6	10.578039	1.746400
56	10.867885	2.399859
53	11.005039	1.900949
47	11.554290	2.156569
61	11.907689	1.542159
51	12.887440	1.828810

Grand Mean = 9.4406

Mean Square Error = 1.314

Bartlett Coefficient = 33.252

$\chi^2(.95; 22) = 33.92$

1. The observations for each factor level are random observations from corresponding probability distributions and are independent of the observations for any other factor level.

2. Each factor level of response has a probability distribution which is normal.

3. All factor level distributions have the same variance.

Test for equal means. Analysis of variance convincingly rejected the hypothesis that mean rates of return among firms within groups were equal (see Table B.4). The Schieffé Multiple Comparison Test was then used to test the hypothesis

$$H_o: \; L_k = \sum_{j=1}^{2} c_j \mu_j(R) = 0 \tag{B.13}$$

where

L_k = the kth pairwise difference in the family of differences within group L or H

$\mu_j(R)$ = the mean rate of return for firm j, a member of group L or H

c_j = a coefficient such that $c_1 = 1$ and $c_2 = -1$.

An unbiased estimate of L_k is

$$\hat{L} = \sum_{j=1}^{2} c_j \overline{R}_j \tag{B.14}$$

where $\overline{R}_j$ is the sample average rate of return for firm j.

An unbiased estimate of the variance of $\hat{L}_k$ is

$$s^2(\hat{L}) = \text{MSE} \sum_{j=1}^{2} \frac{c_j^2}{n_j} \tag{B.15}$$

Table B.4

Analysis of Variance Results for $\left[\frac{EBIT}{A} \text{ x } 100 \right]$

Group L

Source of Variation	Sum of Squares	df	Mean Square
Between firms	532.922	24	22.205
Within firms	216.949	475	.457
Total	749.871	499	

F* = 48.617

F(.99; 24, 499) = 1.79

Group H

Source of Variation	Sum of Squares	df	Mean Square
Between firms	1037.488	22	47.159
Within firms	574.215	437	1.314
Total	1611.703	459	

F* = 35.889

F(.99; 24, 459) = 1.84

Scheffé has shown that the probability is $(1 - \alpha)$ that all statements of the type

$$\hat{L} - S \cdot s(\hat{L}) \leq L \leq \hat{L} + S \cdot s(\hat{L}) \qquad (B.16)$$

are correct simultaneously where $\hat{L}$ and $s^2(\hat{L})$ are given by (B.14) and (B.15) respectively. S is defined as

$$S = [(r - 1)F(1 - \alpha; r - 1, N - r)] \qquad (B.17)$$

where

r = the number of factor levels

α = level of significance

N = total number of observations over all factor levels

F = critical value of the F distribution

Thus, the null hypothesis (see equation (B.13)) cannot be rejected so long as

$$\left|\hat{L}_k\right| \leq S \cdot s(\hat{L}) \qquad (B.18)$$

Clustering firms. Figures B.3 and B.4 identify those firms in groups L and H for which the absolute difference between mean rates of return is less than or equal to the Scheffé critical value. For example, the absolute difference between the mean rates of return for firms 28 and 37 (see Table B.2) is

$$\left|\bar{R}_{28} - \bar{R}_{37}\right| = \left|7.848 - 8.366\right| = .488$$

which is less than the Scheffé critical value (1.401) for firms in group L. The Scheffé critical value is unique to each pairwise

Firm No.	8	19	16	17	21	33	23	26	31	28	36	24	15	13	22	62	37	57	65	41	12	55	58	64	66
8	X	X	X	X	X																				
19	X	X	X	X	X	X	X	X	X	X	X	X													
16			X	X	X	X	X	X	X	X	X	X	X	X											
17				X	X	X	X	X	X	X	X	X	X	X	X	X	X								
21					X	X	X	X	X	X	X	X	X	X	X	X	X	X							
33						X	X	X	X	X	X	X	X	X	X	X	X	X	X						
23							X	X	X	X	X	X	X	X	X	X	X	X	X						
26								X	X	X	X	X	X	X	X	X	X	X	X						
31									X	X	X	X	X	X	X	X	X	X	X						
28										X	X	X	X	X	X	X	X	X	X	X	X	X			
36											X	X	X	X	X	X	X	X	X	X	X	X			
24												X	X	X	X	X	X	X	X	X	X	X			
15													X	X	X	X	X	X	X	X	X	X	X		
13														X	X	X	X	X	X	X	X	X	X		
22															X	X	X	X	X	X	X	X	X		
62																X	X	X	X	X	X	X	X		
37																	X	X	X	X	X	X	X		
57																		X	X	X	X	X	X		
65																			X	X	X	X	X	X	X
41																				X	X	X	X	X	X
12																					X	X	X	X	X
55																						X	X	X	X
58																							X	X	X
64																								X	X
66																									X

(1)
(2)
(3)
(4)

Figure B.3

Equal Mean Rates of Return, Group L Scheffé Critical Value = 1.4007

Firm No.	35	1	52	44	27	4	7	14	67	34	10	29	30	11	39	68	59	6	56	53	47	61	51
35	X	X	X	X	X	X	X	X	X	X	X												
1		X	X	X	X	X	X	X	X	X	X	X											
52			X	X	X	X	X	X	X	X	X	X	X	X	X	X							
44				X	X	X	X	X	X	X	X	X	X	X	X	X	X						
27					X	X	X	X	X	X	X	X	X	X	X	X	X						
4						X	X	X	X	X	X	X	X	X	X	X	X	X					
7							X	X	X	X	X	X	X	X	X	X	X	X					
14								X	X	X	X	X	X	X	X	X	X	X					
67									X	X	X	X	X	X	X	X	X	X	X	X			
34										X	X	X	X	X	X	X	X	X	X	X			
10											X	X	X	X	X	X	X	X	X	X			
29												X	X	X	X	X	X	X	X	X	X		
30													X	X	X	X	X	X	X	X	X		
11														X	X	X	X	X	X	X	X	X	
39															X	X	X	X	X	X	X	X	
68																X	X	X	X	X	X	X	
59																	X	X	X	X	X	X	
6																		X	X	X	X	X	
56																			X	X	X	X	X
53																				X	X	X	X
47																					X	X	X
61																						X	X
51																							X

Figure B.4

Equal Mean Rates of Return, Group H Scheffé Critical Value = 2.3063 comparison. However, it is identical in this analysis for all comparisons within each risk class because the number of observations for all firms are identical. All comparisons resulting in a mean rate of return difference less than the Scheffé critical value are indicated by an X in the appropriate cell, e.g., cell (28, 37) in Figure B.3. Because the multiple comparisons test was conducted on increasing mean rates of return, clusters of firms were formed with mean rates of return within clusters being not significantly different.

Several clusters of firms can be observed in each of the figures. Many of the clusters overlap. For instance, in Figure B.3 there are three clusters of fourteen firms each; e.g., cluster (1) includes firms 17, 21, 33, . . ., and 37. It can be seen that mean rates of return for all but one of the firms in cluster (1) are not significantly different from the mean rates of return for the fourteen firms in cluster (2), and the mean rates of return for all but one of the firms in cluster (2) are not significantly different from the mean rates of return for the fourteen firms in cluster (3). In other words, cluster (3) includes 13/14 of cluster (2) and 12/14 of cluster (1). On the other hand, clusters (1) and (2) do not overlap at all with cluster (4), while firm number 65 is common to both cluster (3) and cluster (4). A similar clustering of firms, many of them overlapping, can be seen for group H firms, with clusters (1) and (2) consisting of fourteen firms each.

It is possible to select from the low and high variance groups many different clusters of firms with identical rate of return distributions. Cluster (1) from group L (hereafter L1) and cluster (2) from group H (hereafter H2) were selected for the following reasons. One, it is desirable to have as many observations as possible for the cross-section regressions, and two, clusters L1 and H2 provide a slightly greater difference between risk classes than do other clusters of the same size. Appendix C lists the firms contained in risk classes L1 and H2.

Rate of Return Correlations Between Firms

Tables B.5 and B.6 contain correlation coefficients for the rates of return between pairs of firms within each risk class. The temporal characteristics of firms' rates of return vary considerably. As noted in the theoretical development of equal risk classes (see Chapter II), the most rigorous condition requires the returns of all firms within a defined class to be perfectly, positively correlated. To the extent that investors have a preference for the timing of returns, as well as the mean and variance of the distribution, they will not view firms within either group as being equal in business risk.

Table B.5

Group L Rate of Return Correlations

	17	21	33	23	26	31	28	36	24	15	13	22	62	37
17	1.00	.114	.133	.155	.549	.378	.381	.033	.252	.748	.564	.223	-.282	.595
21		1.00	.281	-.108	.306	.094	.381	.528	.225	.350	-.447	.336	-.504	-.069
33			1.00	.096	.417	.263	-.098	.596	.547	.467	.108	.629	-.270	.026
23				1.00	-.165	.476	.159	.322	.426	.182	.082	.290	.161	.336
26					1.00	.337	.117	.231	.351	.686	.248	.342	-.604	.307
31						1.00	-.028	.232	.639	.466	.415	.686	-.101	.469
28							1.00	-.111	.126	.509	-.095	-.006	-.500	.315
36								1.00	.457	.331	-.246	.479	-.184	.093
24									1.00	.676	.416	.652	-.288	.671
15										1.00	.437	.484	-.519	.682
13											1.00	.153	.225	.623
22												1.00	-.326	.198
62													1.00	-.187
37														1.00

Table B.6

Group H Rate of Return Correlations

	44	27	4	7	14	67	34	10	29	30	11	39	68	59
44	1.00	.588	.402	.888	.000	.364	.589	.797	.697	.200	.736	.355	.348	.648
27		1.00	-.056	.607	.356	.382	.680	.638	.493	.558	.525	.319	-.096	.714
4			1.00	.470	-.311	-.51	-.165	.134	.603	-.370	-.108	.010	.782	.271
7				1.00	.139	.363	.702	.825	.759	.357	.709	.371	.224	.654
14					1.00	.178	.622	.299	.046	.270	.156	-.116	-.524	.433
67						1.00	.622	.561	-.009	.688	.845	.449	-.515	.071
34							1.00	.795	.465	.683	.754	.321	-.323	.656
10								1.00	.670	.458	.808	.261	.125	.608
29									1.00	.273	.406	.483	.480	.758
30										1.00	.590	.643	-.480	.389
11											1.00	.515	-.155	.430
39												1.00	-.126	.406
68													1.00	.101
59														1.00

APPENDIX C

RISK CLASS L1

Firm Number	Firm Name
17	Orange and Rockland Utilities
21	Potomac Electric Power
33	Utah Power and Light
23	Public Service Co. of NH
26	Rochester Gas and Electric
31	Union Electric
28	Southern California Edison Co.
36	Atlantic City Electric
24	Public Service Electric and Gas
15	New York State Electric and Gas
13	Long Island Lighting
22	Public Service Co. of Colorado
62	Montana-Dakota Utilities
37	Boston Edison Co.

RISK CLASS H2

Firm Number	Firm Name
44	Detroit Edison Co.
27	San Diego Gas and Electric
4	Central Maine Power
7	Columbus and Southern Ohio
14	Nevada Power Co.
67	Philadelphia Electric Co.
34	Virginia Electric and Power
10	Delmarva Power and Light
29	Toledo Edison Co.
30	Tucson Gas and Electric
11	Duquesne Light Co.
39	Central Illinois Light
68	Public Service Co. of Indiana
59	Kansas Gas and Electric

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BIOGRAPHICAL SKETCH

The author was born in Orange, California, on May 6, 1942. During his childhood he moved with his family of four children to several cities in the United States and Europe as his father, a Naval officer, was transferred. His high school education was completed in Madrid, Spain, followed by three years of college at Virginia Polytechnic Institute and State University. The next six years were spent in the United States Marine Corps, the last four serving as an infantry officer. He married Katherine Shawhan of Miami, Florida, in January 1968, and in August 1969 a son, John Jr., was born. The author was an undergraduate and graduate student at Virginia Polytechnic Institute and State University and a graduate student in the Department of Finance at the University of Florida from 1970 to 1976. Since 1976 he has held the positions of Assistant Professor of Finance at Texas Tech University and Virginia Polytechnic Institute and State University. A second son, Benjamin, was born in November 1978.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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This dissertation was submitted to the Graduate Faculty of the Department of Finance in the College of Business Administration and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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